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RADC-TR-67-420, Volume II
Final Report

ACCELERATED TESTING TECHNOLOGY

Volume II Handbook of Accelerated Life Testing Methods

W. Yurkowsky
et al

Hughes Aircraft Company

TECHNICAL REPORT NO. RADC-TR-67-420
November 1967

Rome Air Development Center
Air Force Systems Command
Griffiss Air Force Base, New York

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FOREWORD

This final report was prepared by W. Yurkowsky, R.E. Schafer and J.M. Finkelstein of Hughes Aircraft Company, Ground Systems Group, Fullerton, California, under Contract AF30(602)-4046, project number 5519, task number 551902. Secondary report numbers FR 67-16-157, FR 67-16-185, reporting period covered February 1966 to July 1967. RADC project engineer Donald W. Fulton (EMERR).

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This report has been reviewed and is approved.

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ABSTRACT

This final report is a result of a study performed for RADC under Contract AF 30(602) -4046. The purpose of the study was to survey, document and report on the available methods of reducing reliability test times and test costs. The detailed results of this study have been, as required by the contract, produced in an "Accelerated Life Test Handbook." The methods of reducing test time/costs available are included in quite some step by step procedural detail in the "Handbook." For this reason, this final report is of somewhat a supplemental nature (to the "Handbook"). A serious reader of this report may well find the "Handbook" of interest also. The methods surveyed and written up as possibilities for reducing reliability testing time/costs were classified as:

1) Accelerated Life Test (ALT) Methods (electronic, electromechanical and mechanical parts).

   Important ALT's: Step stress tests, Inverse power rule test, and Arrhenius and Eyring models.

2) More Powerful Statistical Methods

   Important Methods: Bayes tests, distribution free and distribution dependent tests.

Also considered is

3) The multiple modes of failure problem.

In this report each of the above three classifications is described in some detail with respect to

1) present state of art
2) recent advances
3) shortcomings and recommendations for future advancement.

It was found that while there has been a good deal of work written on the problem of reducing reliability test time/costs, only a fraction of it is of excellent quality and that more research is required particularly in the area of ALT validation and algorithms. In the area of Bayes methods, more work is required on prior distributions.

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1.0 INTRODUCTION TO ALT HANDBOOK

1.1 The Reliability Problem

Perhaps the most serious (and persistent) reliability problem facing Industry and the Government is that: relatively long and costly tests are required to demonstrate and/or estimate the reliability needed in the present day "environment." This environment is characterized by

1) High reliability requirements.
2) Tight delivery schedules.
3) Low dollar budgets.

Without intending to belabor this well-known truism, it is worthwhile to see why long test times occur by way of an example. The consumer customarily says I need an MTBF, say θ_1, demonstrated to a 90% confidence. A moderately sharp producer soon learns that to fulfill this requirement he must design (and pass) a reliability test which gives an MTBF of size θ_1 only 10% (100%-90%) chance of passing the test. The producer then knows if he passes such a test he has demonstrated an MTBF of θ_1 to a 90% confidence level. This is a statistical fact of life. However, another fact of life is that the producer, in order to stay in business, needs a probability of passing this test of somewhere around 90%. The producer must then design his "part" so that its MTBF exceeds θ_1, say he designs it to $\theta_0 > \theta_1$. If he designs it so that θ_0 exceeds θ_1 by a slight amount only he must face up to long test times (a statistical fact of life) and 2) and 3) above come into play. If he designs the "part" so that $\theta_0 > \theta_1$ by a large amount he runs into 3) above and even state of the art limitations on reliability. To make matters even worse, the sample size in life testing is essentially the number of failures that occur and this is a function (for fixed test time) of the true "part" MTBF.

1.2 Purpose of the Handbook

The reliability problem outlined above can never be solved completely, (e.g., test time equal to zero) but there are a number of methods in the reliability literature which if employed in the proper situations can help alleviate the problem a great deal. It is these methods that this handbook seeks to present. In other words, the purpose of this handbook is: to present valid (and some unvalidated but very promising) methods of reducing the time/costs of demonstrating and/or estimating reliability.

1.3 Classification of Methods of Reducing Test Time and Costs

There seem to be two broad classes of reducing test time/costs:

1) Accelerated life tests.
2) More powerful statistical methods.

In a certain sense 1) above falls into the category 2) above but the division is quite natural and is used in this handbook.

One of the great problems in reliability work is the time delay between the development of good methods of test time/cost reduction and their general acceptance and use. Another problem is the relatively large amount of literature making it difficult for an Engineer to keep up-to-date. Finally, it is often difficult for the practicing Engineer, with many other problems at hand, to properly evaluate and understand the methods proposed. This handbook seeks to solve some of these problems. For this reason, the handbook has the following sort of character:

1) It is based on a thorough search of the literature (see Bibliography).

2) It presents the methods in cookbook form with little or no proofs.

3) The criteria for the inclusion of a method have been rather stringent. The one exception to this is the case where some very promising, but as yet unvalidated, accelerated life tests have been included. These are clearly labeled.

1.4 Accelerated Life Test Methods

The accelerated life test methodology (hereafter referred to as ALT) has two outstanding characteristics:

1) It is voluminous.
2) The methods are mostly unvalidated.

The second remark requires a little explanation, particularly, the word validated. The definition of an ALT used in this handbook is:

> An ALT is a test run (usually on a part) at operating/environmental conditions which provides reduction in test costs over normal operating/environmental conditions and which provides an algorithm for extrapolating the reliability observed at accelerated conditions to reliability observed at normal conditions.

It is in this "extrapolation algorithm" that most of the shortcomings of ALT occur. First, and very regrettably, the method of extrapolation is often only vaguely given, if given at all. Secondly, the extrapolation method, when given, was often not empirically validated by sound statistical methods. Finally, a physical model which in some sense "supports" the statistically validated extrapolation is often missing. The physical model which supports the extrapolation method is a sort of insurance. As is well known, an empirically derived relationship carries no guarantees that the relationship will persist. The physical model when it exists can be used to explain the empirical results and provide indications of whether the relationship (extrapolation algorithm) can be expected to persist. The physical model can also be used to provide insight as to whether the particular ALT (usually derived on a particular part or specimen) can be used on other parts of the same family.

The ideal ALT thus involves:

1) An algorithm (instruction) for converting the reliability data observed at accelerated conditions to reliability data at normal conditions.

2) A statistically sound empirical validation that the algorithm works.

3) A physical model that explains the statistically sound algorithm.

It is regrettable that, had the handbook required all three above, it would have been very small indeed. Basically what has been done is to drop requirement 3) above for validation. Where this has been done, it is pointed out. If the remaining requirements are still too stringent to suit the tastes of the reader, the available "ALT methods" are called out in the Bibliography.

A trip through the available ALT literature quickly displays the complexities of the problems involved in developing an accurate, repeatable, useful and economically feasible tool for the Engineer. The problem, while not easy to solve, is easy to pinpoint: The field of ALT jointly involves the disciplines of 1) electrical/mechanical engineering, 2) physics, 3) mathematics, 4) statistics and 5) reliability engineering. Often one finds a particular ALT being written by an author in one and only one of the above disciplines. The amount of information lost in the translation between disciplines is astonishing. Examples are available but have no place here.

On the other hand, there are many good things in the ALT literature. It is hoped a share of these appear in this handbook.

1.5 More Powerful Statistical Methods

The word powerful, as used in this handbook, is used rather more generally than the usual statistical meaning of power. A statistical method (as it applies to testing and estimation problems in life testing) may be considered more powerful in this handbook because (provided other test characteristics are of the same order):

1) the method reduces test time/sample size required for given confidence in common situations.

2) the method uses all available information and thus provides the correct model. (e.g., Bayes test plans and methods.)

3) the method may save data analysis costs or otherwise simplify testing (e.g., order statistics).

The user of this handbook will find that the More Powerful Statistical Methods section is not a text on statistics or even statistical methods in life testing. Its purpose is to present recent, more modern statistical methods of life testing than those methods which can reasonably be regarded as "common knowledge." The purpose of the section then is to help shorten the time delay between the development of useful methods in the literature and their understanding and acceptance in applications.

There remains in the field of statistical methods in life testing plenty of room for improved statistical methods. Some of these are presented in section 5.0.

1.6 The Multiple Modes of Failure Problems

Perhaps the most jealously guarded truism ALT has to offer is: a given accelerated test can never be valid if the modes of failure change between normal and accelerated conditions. Section 6.0 of this handbook takes a look at this statement in terms of statistical models of failure. Specifically known observed results are evaluated in terms of models for multiple modes of failure. Certain problems are brought to light and discussed.

2.0 IMPROVED VALIDATION METHODS

2.1 Introduction

As previously indicated an ideal accelerated test involves not only statistically valid empirical results but a supporting physical model. The Eyring and Arrhenius models are cases in point. It turns out that when a physical model is available, the empirical results can be used to estimate the parameters of the physical model.

The physical models eventually involve an assumption about the relation between two random variables:

x = random variable denoting time to failure under reference stress.

y = random variable denoting time to failure under accelerated stress.

The Arrhenius and Eyring models often use $y = \tau x$ where τ is a constant, depending on the applied stresses (including the reference stress). The inverse power law uses $x = ky^m$ usually $m > 1$ and k is a constant depending on some stress and perhaps other constants.

Very often (particularly in the original development) the empirical failure distributions at reference and accelerated conditions are available. This makes it possible to use certain results of statistics to verify physical models and estimate the parameters of them. The table on the following page gives the results for some frequently occurring failure distributions.

The use of the results of the table is best seen by examples.

1. In the case exponential/exponential, the Eyring and Arrhenius model is

$$y = \tau x,$$

and the table implies $\tau = \frac{\lambda_x}{\lambda_y}$ and τ may be estimated by estimating λ_x/λ_y.

The table also shows that if 2., 3., 4. $(\beta_x \neq \beta_y)$, 5. $\left(\frac{\sigma_y}{\sigma_x} \neq \frac{\mu_y}{\mu_x}\right)$, 6. $(\sigma_{1\,g\,y} \neq \sigma_{1\,g\,x})$ have been experienced, then the linear transformation $y = \tau x$ cannot occur and any physical model which leads to it is not consistent with the observed data.

FAILURE DISTRIBUTIONS	IF	AND	THEN
1. Exponential/ Exponential	$F(x)=1-e^{-\lambda_x x}$; $x,\lambda_x > 0$	$F(y)=1-e^{-\lambda_y y}$; $y,\lambda_y > 0$	$y=\left(\frac{\lambda_x}{\lambda_y}\right)x$
2. Exponential/ Weibull	$F(x)=1-e^{-\lambda_x x}$	$F(y)=1-e^{-\frac{y^\beta}{\alpha}}$; $\alpha,\beta,y > 0$ $\beta \neq 1$	$y=(\alpha\lambda_x)^{1/\beta}\,x^{1/\beta}$
3. Weibull/ Exponential	$F(x)=1-e^{-\frac{x^\beta}{\alpha}}$; $\beta \neq 1$	$F(y)=1-e^{-\lambda_y y}$	$y=\left(\frac{1}{\lambda_y\alpha}\right)x^\beta$
4. Weibull/ Weibull	$F(x)=1-e^{-\frac{x^{\beta_x}}{\alpha_x}}$; $\beta_x \neq 1$	$F(y)=1-e^{-\frac{y^{\beta_y}}{\alpha_y}}$; $\beta_y \neq 1$	$y=\left(\frac{\alpha_y}{\alpha_x}\right)^{1/\beta_y} x^{\beta_x/\beta_y}$
5. Normal/ Normal	$F(x)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}\,\sigma_x}e^{-\frac{(s-\mu_x)^2}{2\sigma_x^2}}\,ds$	$F(y)=\int_{-\infty}^{y}\frac{1}{\sqrt{2\pi}\,\sigma_y}e^{-\frac{(t-\mu_y)^2}{2\sigma_y^2}}\,dt$	$y=\left(\frac{\sigma_y}{\sigma_x}\right)x-\left[\frac{\sigma_y}{\sigma_x}\mu_x-\mu_y\right]$
6. Log Normal/ Log Normal	log x has normal d.f.	log y has normal d.f.	$y=x^{\frac{\sigma_{\log y}}{\sigma_{\log x}}}\,e^{-\left(\frac{\sigma_{\log y}}{\sigma_{\log x}}\mu_{\log x}-\mu_{\log y}\right)}$
7. Gamma/Gamma	$F(x)=\int_{0}^{x}\frac{\alpha_x^{\lambda_x}}{\Gamma(\lambda_x)}s^{\lambda_x-1}e^{-\alpha_x s}\,ds$	$F(y)=\int_{0}^{y}\frac{\alpha_y^{\lambda_y}}{\Gamma(\lambda_y)}t^{\lambda_y-1}e^{-\alpha_y t}\,dt$	$y=\left(\frac{\alpha_x}{\alpha_y}\right)x$

2. The inverse power rule gives $y = c\, x^{1/m}$ where c is a constant and usually $m > 1$. Unless $m=1$, case 1. above is not consistent with the inverse power rule. Neither are cases 5. and 7. The other cases will work for the inverse power rule and the exponent $\frac{1}{m}$ is readily estimated. For example, take case 4. in the above table; $1/m$ may be estimated by estimating β_x/β_y. The c of $y = cx^{1/m}$ may be estimated by estimating (in case 4.) the quantity

$$\left(\frac{\alpha_y}{\alpha_x}\right)^{1/\beta_y}$$

In summary, whatever form the physical model proposes for $y = f(x)$, the observed failure distributions (at reference and accelerated stress conditions) furnish a means of checking the assumptions on the form of $y = f(x)$ and estimating its parameters if it is correct to do so.

It should be noted that the three column headings (If, And, Then) of the table may be suitably interchanged. For example, the "And" and "Then" headings may be interchanged.

2.2 The Failure Mechanism Approach

In recent years, increasing attention on the part of device physicists is being devoted to methods for determining whether the modes of failure change between normal and high stress test conditions. This approach (Reference 519) is based upon attempts to identify and understand the nature of processes (or mechanisms), in materials, which lead to degradation and failure of devices. In essence, from observations of device response during laboratory stress tests and from a careful, detailed analysis of failed devices, one attempts to identify the dominant mechanism(s) and establish expressions for its time and stress dependencies. A measurable device parameter is then sought which may be used as a sensitive indicator of the dominant failure mechanism. Device failure may then be expressed in terms of changes in device parameters which are explicitly related to the underlying failure mechanism or degradation process. If one can obtain such kinetic expressions for each of the principal contributing failure mechanisms over the stress ranges of interest, then the stress levels at which mechanisms of failure change may, in principle, be determined. The stress range of validity for ALT is then also determined.

The failure mechanism approach is a highly attractive one, since it offers the possibility of a sound physical basis, not only for ALT, but also for device design and improvement, reliability test design and screening techniques, and the extrapolation of reliability data to new device situations. Unfortunately, the approach

is not a simple one in practice, and there are several limitations. It is being applied most extensively to electronic parts, and especially to semiconductor devices.

A failure mechanism, in this context, is a theoretical model devised to explain at the atomic and molecular level the outward manifestation of failure relating to the terminal device behavior, or the failure mode of the device. As a consequence of this fact, several difficulties arise:

1. Theoretical models of failure mechanisms are indeterminate in the sense that they are not unique. They describe processes at the atomic level and hence cannot usually be confirmed by direct observation.

2. Failure mechanisms are quite complex and do not as a general rule occur singly but rather in combination, hence, identification and isolation of them individually is quite difficult.

3. The models are highly idealized, and while they are often used to predict dependencies and thus lead to better understanding of the process involved, quantitative results depend critically upon fine details in the composition and fabrication of a device. The influence of defects, whether intrinsic or process-induced, on the kinetics of failure mechanisms is not well understood.

4. Device failure rate due to a particular failure mechanism does not necessarily have the same dependence on a stress that the failure mechanism does.

5. The degradation process or failure mechanism is not always reflected initially in a change in measurable device parameters.

Despite these difficulties, the failure mechanism approach still constitutes a useful supplement to other validation methods. As with most ALT methods, it lacks rigor, but its principal justification lies in showing that observed time and stress dependencies follow the theoretically expected behavior of a consistent physical model of the failure mechanism.

To date, most applications of the method have utilized the Arrhenius or Eyring formulations as a basis for the extrapolation of failure rate data. When failure mechanisms exhibit an Arrhenius-type temperature dependence, a plot of the logarithm of failure rates is sometimes found to be proportional to reciprocal absolute temperature of an active element of the device. A plot of this relationship yields a straight line over the temperature range where the same failure mechanism applies. The slope of this line then gives an apparent (or empircal) activation energy of the failure mechanism. Changes in the slope of the line may be interpreted as the

result of another mechanism becoming dominant at that temperature and thus determine the range of valid stress limits for ALT. This empirically derived activation energy, under certain conditions (References 321, 328) serves as a principal link between macroscopic changes in device behavior and the underlying atomic and molecular mechanisms in materials. The stepwise procedure in applying a simple Arrhenius model to the analysis of test data on the temperature dependence of device degradation are given in Reference 328. Also considered in Reference 328 are modifications to Arrhenius-Eyring type models to include stresses other than thermal.

While the success of the failure mechanism approach is largely dependent upon the likelihood of obtaining quantitative kinetic expressions for the failure mechanisms over the stress ranges of interest, it should be pointed out that even a conceptual understanding may be quite useful in ALT In the study of failure mechanisms, it is true that one does not escape the basic ALT dilemma, i.e., how to obtain information in a minimum time on a physical process which changes slowly at low stress levels. The important distinction, however, is that a transformation has been made from device behavior to the more basic level of physical (and chemical) processes which are fewer in number and are common to a wide variety of materials and devices. Hence, even if the mechanism has only been identified and qualitatively understood, at stress levels where data is lacking confidence may be derived from experience with the same mechanism in other situations.

Examples of device studies in which the failure mechanism approach has been applied to transistors are found in References 520, 521, 522, and to diodes, in Reference 523.

3.0 ALT METHODS FOR ELECTRONIC AND ELECTROMECHANICAL PARTS

This section and the succeeding one are devoted to a presentation of the present state-of-the-art in the field of accelerated life testing. The original plan was to study the literature and select only fully validated methods for presentation in the handbook. It soon became obvious that the various methods attempted and reported in the literature over the years consisted of exploratory studies for the most part. Seldom was there sufficient space to include all the test results. Seldom was there sufficient time to verify the results by performance of a test program at rated stress levels. The consequence of this is that the literature contains much information on ALT methods which were attempted but none which have been conclusively proven to accurately represent the physical failure laws which govern when parts are operated at stresses more severe than those intended by the part's designer.

To include all of the work which has been done regardless of its degree of success would constitute a history of the subject rather than a handbook of useful methods. On the other hand, the presentation of some unsuccessful attempts is sometimes a way of proving that a given model does not hold.

The central point of interest of an ALT model is that it must contain provisions for extrapolating life estimates at severe stresses to accurate life estimates at rated stresses. Furthermore, to validate the ALT method it must be proved that the results are repeatable and that they can be attributed to certain physical failure phenomena. Most experimenters in the field of ALT assume a statistical failure model (algorithm), perform some tests and present their results. Seldom do they attempt to explain in terms of failure laws the reasons for the observed results. Seldom do they even repeat the experiment to verify that comparable results will be obtained upon replication.

Therefore, the pages that follow contain detailed descriptions of representative work to develop certain ALT models. None have been completely validated. The user of the handbook can begin by seeing if ALT methods have been applied to the parts which interest him. To aid in this, the reports of significant works are arranged alphabetically by generic part. An ALT evaluation procedure was performed on each method. The results of the evaluation are presented in the upper right hand corner of the first page of the description of the ALT method.

The specific points used for evaluation of the validity of the ALT methods are:

1) General acceptance of the statistical and engineering assumptions.

2) Presence of empirical proof that the model works.

3) A statistical model for converting estimates at accelerated stresses to results at rated stresses.

4) A physical model to explain the observed results in terms of failure laws.

An ALT method can be useful even though every one of the four attributes are not known or available. For example, even though the physical model is not known, one can still perform tests based on sound engineering and statistical principles and through the use of the correct statistical model it would be possible to obtain valid results. However, the chances of making a complete and accurate assessment of the results would be reduced if knowledge of the physical model was missing. In a like manner, if all the attributes were known except that the test program was conducted on inaccurate equipments or by methods involving large experimental error, the chances of making accurate ALT predictions would be considerably lessened.

Each ALT method presented in the book since it is keyed to a given part type is based on study by only one or at most just a few researchers. The idea of the handbook was to present the method in a complete enough format to allow a reader to use the method without need of assembling other references. Therefore, the user of the book should be able to first look for the generic part type of interest, then observe the evaluation of the available ALT methods, and in reading the write-up in the handbook use the method without further references. It must be pointed out, however, that many instructions had to be curtailed to prevent writing a text on statistics. For example, in many of the Instructions for Use subsections, it was necessary to say that the failure times were to be plotted on Weibull probability paper. Therefore, it had to be assumed that the user of the book possesses this knowledge. This same situation prevailed in the description of test methods also, as well as all phases of the study of ALT methods.

In each of the ALT methods presented in Sections 3 and 4, there is a subsection titled "Instructions for Use." Usually, the first instruction reads "Take a sample of a size large enough to produce statistically significant results." At first glance "statistically significant" appears to be an ambiguous phrase. Actually, it has a rather precise meaning as will be seen in the following discussion.

The major purpose of life tests is the estimation (point and interval) of population parameters (e.g., β of the Weibull, mean, percentiles, etc.) from sample data. For this purpose "estimators" are used. There are many kinds of estimators with varying properties but they all have two things in common

1) They are computed from sample (life test) data.

2) They generally differ, because of sampling error, from the the parameters they are estimating.

Statistically significant refers to 2) above. It means that: <u>in terms of the particular situation at hand</u> (e.g., accuracy desired, purpose of test, etc.) the sample size selected is large enough so that the sampling error involved in estimating the parameter is "small." The magnitude assigned to "small" depends entirely on the individual situation.

Consider an example of a non-statistically significant sample size. Surely, it can be said that, without any data (i.e., n=0), an $0 \leq MTBF \leq \infty$. This is an interval estimate of MTBF but hardly of statistical significance. If money/time is the overriding factor in sample size, then the term statistical significance does not apply. If money/time is not the overriding factor, then statistical significance means:

> With high probability the estimate differs by no more than $\pm$ x% of the parameter being estimated.

Once having assigned numbers to "high probability" and x there are available standard methods of finding the sample size, n, for various estimating situations.

In the event that the user of the handbook does find it desirable to refer to the publication from which the method in the handbook was derived, the same symbols and definitions used by the author have been maintained in the preparation of the handbook.

The remainder of Section 3 of the handbook contains ALT methods for electronic parts. Section 4 is devoted to the information available for ALT methods on mechanical parts.

3.1 PART NAME AND DESCRIPTION:

Capacitors, Ceramic (MIL C-30D and MIL C-11015)
Capacitors, Mica (CD-15)
Capacitors, Paper and Film (Qualified under MIL C-14157B).

ALT Evaluation	
Assumptions Generally Accepted	
Empirical Proof	
Algorithm	X
Physical Model	

SOURCE: These ALT methods were published in the Cornell-Dubilier Reliability Manual dated March 1963. (Reference 504.)

PURPOSE OF TEST: To calculate the failure rate of capacitors operated at rated voltage and temperature levels based on overstress test results and the assumption of an exponential distribution of failure times.

DEGREE OF VALIDATION

The inverse power rule has been used in the literature by many authors to estimate the life of capacitors of many varieties. The major problem encountered by most was in the accurate estimation of the power factor. A slight error in the estimation of n, results in a very large error in the K factor. No information is presented to fortify the estimates of the n values presented for the various capacitor types. Other publications indicate that the power factor for the voltage effect on life changes as the temperature changes and yet no provision is made for this situation in this method. The assumption of a constant failure rate is also controversial based on other publications. No empirical proof is furnished to validate the method.

DESCRIPTION OF TEST METHOD: No details of the test equipments, methods, or sample sizes used are presented.

SUMMARY OF RESULTS

The model used for extrapolation is the inverse power law for overstress voltage and an acceleration factor for temperature. Each of the three types of capacitors covered in this report are treated with slight variations although the general methodology is the same. The general method for voltage acceleration results in the calculation of a K factor based on the following relationship:

$$K_v = \left(\frac{V_1}{V_2}\right)^n \qquad (3.1.1)$$

where K_v = voltage acceleration constant

V_1 = a given overstress voltage

V_2 = a given rated voltage (or any voltage lower than V_1)

n = power exponent

The use of this formula is illustrated if a ceramic capacitor is rated at 250 volts and is put on accelerated test at 500 volts. For this type of capacitor the power exponent is given as 3. Therefore, the voltage acceleration factor would be calculated from the above formula as follows:

$$K_v = \left(\frac{500}{250}\right)^3 = 8$$

The failure rate calculated from the accelerated test data (500 volts) would be divided by K_v to yield an estimate of the failure rate at 250 volt operation. These calculations are based on the assumption of a constant failure rate.

If temperature is used as an accelerating stress the K factor for the failure rate is calculated from the following formula:

$$K_t = 2^{\left(\frac{T_1 - T_2}{T_3}\right)} \qquad (3.1.2)$$

where: K_t = temperature acceleration constant

T_1 = accelerated temperature in °C

T_2 = rated temperature (or some temperature lower than T_1) in °C.

T_3 = a factor based on the magnitudes of T_1 and T_2. (It is based on the assumption that below 85°C, part life is doubled for every 20°C decrease in temperature. From 85°C to 125°C life time is halved for every 10°C rise in temperature. Above 125°C life time is halved for every 8°C rise in temperature.)

Therefore, if an accelerated test is to be run at 200°C and one wants to adjust the failure rate obtained at 200°C to 85°C the following calculations would be made:

$$K_t = 2^{\left(\frac{200-125}{8}\right)} \times 2^{\left(\frac{125-85}{10}\right)}$$

$$K_t = 2^{13.4} = 9192$$

The failure rate calculated from the 200°C data would then be divided by K_t to get the equivalent failure rate at 85°C. This again is based on the assumption of an exponential distribution of failure times.

The same formulae for temperature acceleration and voltage acceleration are used for the other types of capacitors mentioned in Reference 504 but the power factors and temperature factors vary accordingly for each different variety.

INSTRUCTIONS FOR USE

1. Select a random sample of parts with the sample size large enough to provide estimates that are statistically significant(see Sec. 3.0).

2. Test the parts to failure at a voltage level higher than manufacturer's rated and/or at a temperature greater than 85°C.

3. Calculate the failure rate from the results of the accelerated tests using the assumption that failure times are distributed exponentially.

4. If voltage was used as the accelerating stress calculate the voltage acceleration constant (K_v) with formula 3.1.1.

5. The following table gives levels of power exponents for various types of capacitors:

Capacitor Type	n
Ceramic	3
Mica	5
Paper and Film	9

6. Adjust the failure rate calculated from the accelerated test data by dividing it by the K_v from Step 4.

7. If temperature was used as the accelerating stress, use formula 3.1.2 to calculate the temperature acceleration constant (K_t). For T_3 in the formula use the following values depending on the capacitor type:

Capacitor Type and T Temperature Range	T_3
Ceramic (<85° C)	8
Ceramic (85°-125°C)	10
Ceramic (>125°C)	20
Mica	20
Paper and Film	15

8. Adjust the failure rate calculated from the accelerated test data by dividing it by the K_t from Step 7.

LIMITATIONS/RANGE OF APPLICABILITY

Since this ALT method assumes a knowledge of the value of n in the voltage acceleration constant formula, the use of the method must be precluded by a significant amount of life test data at both rated and accelerated voltages. This same requirement applies to the temperature acceleration formula for the factor T_3.

The method of applying the acceleration constants is based on the assumption of a failure rate that is constant with time. Hence, before the method can be successfully used this characteristic of the parts under test should be a proven fact.

The exact n values and T_3 values are applicable only to the parts produced by Cornell-Dubilier.

REFERENCES: 504

3.2 PART NAME AND DESCRIPTION: Capacitors, Mica

ALT Evaluation	
Assumptions Generally Accepted	
Empirical Proof	
Algorithm	X
Physical Model	

The mica capacitors tested in the development of this ALT method were rated at 510 pf, 300 volts DC, and 125°C.

SOURCE: The results are given in a 1965 IEEE Transactions paper by H. S. Endicott, B. D. Hatch and R. G. Sohmer entitled "Application of the Eyring Model to Capacitor Aging Data." (Reference 111.)

PURPOSE OF TEST: To develop methods for enhancing the use of the inverse power rule in accelerated life testing theory. The relationship between constant stress and progressive stress tests is developed from the Eyring Model. Several methods are presented for determining n, the power exponent in the inverse power rule model.

DEGREE OF VALIDATION

The methodology presented for use of the inverse power rule as an ALT model is very well developed. The rules for determining n, the power exponent result in substantial agreement between the various methods. However, only three points were available for fitting the Eyring Model. Although a straight line was fit through the points this is hardly enough data to validate the physical model. The data for mica capacitors from Reference 113 suffered from certain experimental errors which are discussed at greater length in that paper. The major facts validated from the data are that different temperature levels result in differing values of n. The very important point of whether an n of 9 at 140°C yields valid results can be proved by additional tests.

DESCRIPTION OF TEST METHOD

Test data from another source, Reference 113, were used covering both constant and progressive stress tests. Several analysis methods were utilized to operate on this data in order to achieve the program objectives.

SUMMARY OF RESULTS

The inverse power rule model is developed using the Eyring Model. It results in the following relationship between life and voltage stress when applied at a constant level or at progressively increasing levels.

$$t_c = \frac{1}{n+1}\left(\frac{r}{V_c}\right)^n t_p^{(n+1)} \qquad (3.2.1)$$

where t'_c = equivalent mean life at rated constant voltage V_c

r = rate of increase of voltage in progressive stress test

t_p = mean life in progressive stress test

n = power exponent

The problem in using the above relationship for an ALT method is that n must be known. The methods given for finding n are as follows

1. If constant voltage tests are used, plot the desired statistic of failure times on the abscissa and the voltages as ordinates of log-log paper. Fit a straight line through the plotted points and it will have a slope of $(-1/n)$. Solve for n.
2. If progressive voltage tests are used, solve for n by the same method described in Step 1 above.
3. If groups of mica capacitors are tested at both steady voltage and progressive voltage the value of n may be found by plotting on log-log paper the ordered times to failure of parts tested at constant voltage on the abscissa versus the ordered failure times for corresponding parts tested at increasing voltage stress as ordinates. The slope of the line of best fit through these points will be $\left(\frac{1}{n+1}\right)$.
4. If the failure times at both constant and progressive stress are each distributed according to the Weibull distribution, then the ratio of their shape parameters gives the following relationship for solving for n:

$$\frac{\beta_p}{\beta_c} = n+1 \qquad (3.2.2)$$

where β_p = the Weibull shape parameter from progressive stress tests

β_c = the Weibull shape parameter from constant stress tests.

INSTRUCTIONS FOR USE

1. Select a group of samples of mica capacitors large enough to yield results which are statistically valid.

2. Test the parts to failure at either several levels of constant voltage, several levels of progressive voltage, or some levels at constant and some progressive.

3. Solve for n per the instructions in the previous section on Summary of Results.

4. Using the known value of n, conduct all future tests at accelerated stress levels and use one of the following relationships to convert to equivalent life at rated voltage:

$$t_R = t_A \left(\frac{V_A}{V_R}\right)^n \qquad (3.2.3)$$

for a constant voltage tests where

t_R = mean life at rated voltage V_R

t_A = mean life at accelerated constant voltage V_A

n = power exponent

The above formula can be used for constant voltage tests. For progressive voltage tests use formula 3.2.1.

LIMITATIONS/RANGE OF APPLICABILITY

The theories developed liberalize very completely the procedure required for finding n. One can use constant stress tests, progressive stress tests or a combination of them if they are all conducted at the same temperature. The important consideration must be that everything that is possible to increase the accuracy of the estimate of n should be done. This is important because an error in this value results in a much larger error in estimating life at rated stress. Consequently, sample size, accuracy of test equipment and accuracy of data analysis methods are all extremely important.

REFERENCES

111 and 113.

3.3 PART NAME AND DESCRIPTION: Capacitors, Mica

ALT Evaluation	
Assumptions Generally Accepted	___
Empirical Proof	___
Algorithm	X
Physical Model	___

These parts were rated at 510 pf ± 5%, 300 d.c. WV 125°C, casing D, unscreened. The parts were stored at 74°F ± 2° and 50% relative humidity from the time they were received until they were put on test.

SOURCE: The results describing this ALT method for use on mica capacitors are given in Reference 113 by H. S. Endicott and J. A. Zoellner in January 1961.

PURPOSE OF TEST: The purpose of the test was to investigate methods of estimating n, the power exponent which must be known in order to use the inverse power model for accelerated life testing. The study verified that n is constant over given ranges of voltage and that it varies with temperature. Also, a central objective was the comparison of progressive and constant voltage tests.

DEGREE OF VALIDATION

The authors do not claim validation of this ALT method although portions of their data looked promising. They encountered an effect apparently due to storage age which must be considered. There were test errors introduced due to inaccuracies in the rates of voltage rise in the progressive stress tests. Only linear fits were tried in calculating regression lines. While the values of n were generally constant throughout the voltage ranges studied, the n value decreased as temperatures increased. This point was mentioned by the authors as seeming inconsistent with the working hypothesis of the cumulative damage model.

DESCRIPTION OF TEST METHOD

Figure 3.3.1 is a schematic diagram of the equipment showing the circuits used for the steady stress power supplies and associated controls. The progressive stress power supplies differed only in having synchronous motor driven variable ratio autotransformers.

Voltage was continuously monitored by a Non Linear Systems digital voltmeter and a printed record made at ten minute intervals by means of a Clary printer. A Veeder-Root counter driven by a 1 rpm synchronous motor provided a printed record of elapsed time through its electrical readout.

The capacitors were individually fused on racks holding 50 each. When a failure occurred a relay in series with a group of racks operated to cause a printed record to be made of the circuit number and the elapsed time. The resistance of the fuses was sufficient to prevent surges on adjacent capacitors.

Two thermocouples were mounted on each capacitor rack. The readings were averaged to obtain the actual temperature of a group of capacitors. Approximately 12,000 capacitors were tested to failure in both steady and progressive stress tests. Five steady stress voltages and 3 steady stress temperatures were employed as shown in the following table:

		Test Voltage (Steady Stress)				
		1000	2000	3000	4000	5000
Test	125			X	X	X
Temperature°C	147		X	X	X	
	200	X	X	X	X	

Two hundred and fifty parts were tested at each set of conditions denoted by an X in the table. Progressive voltage tests were performed at .08, 2.2, and 7.0 volts/minute at the three test temperatures and with sample sizes of 250 each.

SUMMARY OF RESULTS

The entire study was based on a working hypothesis that stated that at a fixed temperature and given chemical state the total accumulated damage was defined by the following relationship:

$$\text{Total accumulated damage} = C\int_0^T \left[V(t)\right]^n dt \qquad (3.3.1)$$

where $V(t)$ = voltage applied at time t

n = power exponent of the inverse power rule

C = a constant

Formula 3.3.1 for the case of constant voltage stress becomes

$$\text{Steady Stress Damage to Time } T = CV_s^n T \qquad (3.3.2)$$

Formula 3.3.1 for the case of progressive voltage stress becomes:

$$\text{Progressive Stress Damage to Time } T = \frac{C\lambda^n T^{n+1}}{n+1} \qquad (3.3.3)$$

where λ = uniform rate of rise of voltage from 0 in volts/unit time.

The working hypothesis then theorizes that damage to time T at constant and progressive stress are related by setting formulae 3.3.2 and 3.3.3 equal to each other. The C's cancel out and the following relationship results:

$$T = \frac{\lambda^n \tau^{n+1}}{V_s^n (n+1)} \qquad (3.3.4)$$

The use of this model as the inverse power rule then says that an equivalent steady stress failure time T at constant voltage V_s can be estimated from any progressive stress test with voltage uniformly increased at the rate λ using progressive stress failure time τ if n is known.

The problem of solving for n and verifying its constancy over a given range of voltages was the subject of the tests performed in Reference 113. The estimates of the power exponent n were as follows:

Steady Stress Condition	Temperature °C	n
No Prior Treatment	125	11.4
	147	10.8
	200	6.0
Baked 24 hours before test	125	12.6

The n values were found to be constant over the voltage ranges studied and varied with temperature.

Agreement of the test results and the working hypothesis were attempted by assuming values of n such as 6 or 7, solving Equation 3.3.4 for T and then comparing these calculated values with observed values from the steady stress tests using χ^2 tests to check for significance. The results obtained did not uniformly accept the working hypothesis. This may be due to the fact that the model does not fit or due to several problems encountered in the performance of the test program.

The solution for n was accomplished by converting Equation 3.3.4 to its logarithmic form as follows:

$$\log T_i = \left[n \log \frac{\lambda}{V_s} - \log (n+1) \right] + (n+1) \log \tau_i \qquad (3.3.5)$$

The subscript i refers to the ordered ranks of the failure times at both steady and progressive stress tests. From the above formula it was found that on log-log paper the plot of ordered steady stress failure times versus ordered progressive stress failure times yielded a straight line with slope n+1.

A solution for n could also be made from two or more sets steady stress test results or from two or more sets of progressive stress test results by plotting the log of voltage against log of life. The slope of the straight line fitted through the data will be $-\frac{1}{n}$.

INSTRUCTIONS FOR USE

The following is a step by step instruction for using this ALT method on these parts:

1. Select large enough samples to yield statistically significant results.

2. Test the parts to failure at either several levels of constant voltage stress, several levels of progressive voltage stress, or test some at steady stress and some at progressive stress.

3. If constant stress is used, solve for n by plotting logarithm of voltage versus logarithm of life (median or mean). If the model fits, the data will plot as a straight line with slope $-\frac{1}{n}$.

4. If several levels of progressive stress are used, repeat Step 3 and again solve for n.

5. If tests are run at both progressive and constant stress levels, solve for n with formula 3.3.5. The failure times at both progressive and steady stress times are ordered and their logarithms are plotted (i.e., log T_1 vs log τ_1, etc.). If these data plot as a straight line, it can be assumed that the model fits and the line will have slope n+1.

6. Once n has been found, accelerated tests can be run at either steady or progressive voltages and the working hypothesis for the inverse power rule used for estimating mean life at rated voltage stress.

LIMITATIONS/RANGE OF APPLICABILITY

This study was conducted at many levels of steady and progressive voltage stress and at three temperatures. The authors had some measure of success in studying the model but did not claim it was the proper cumulative damage model. (See section on Degree of Validation.)

REFERENCES: 113.

72161-25

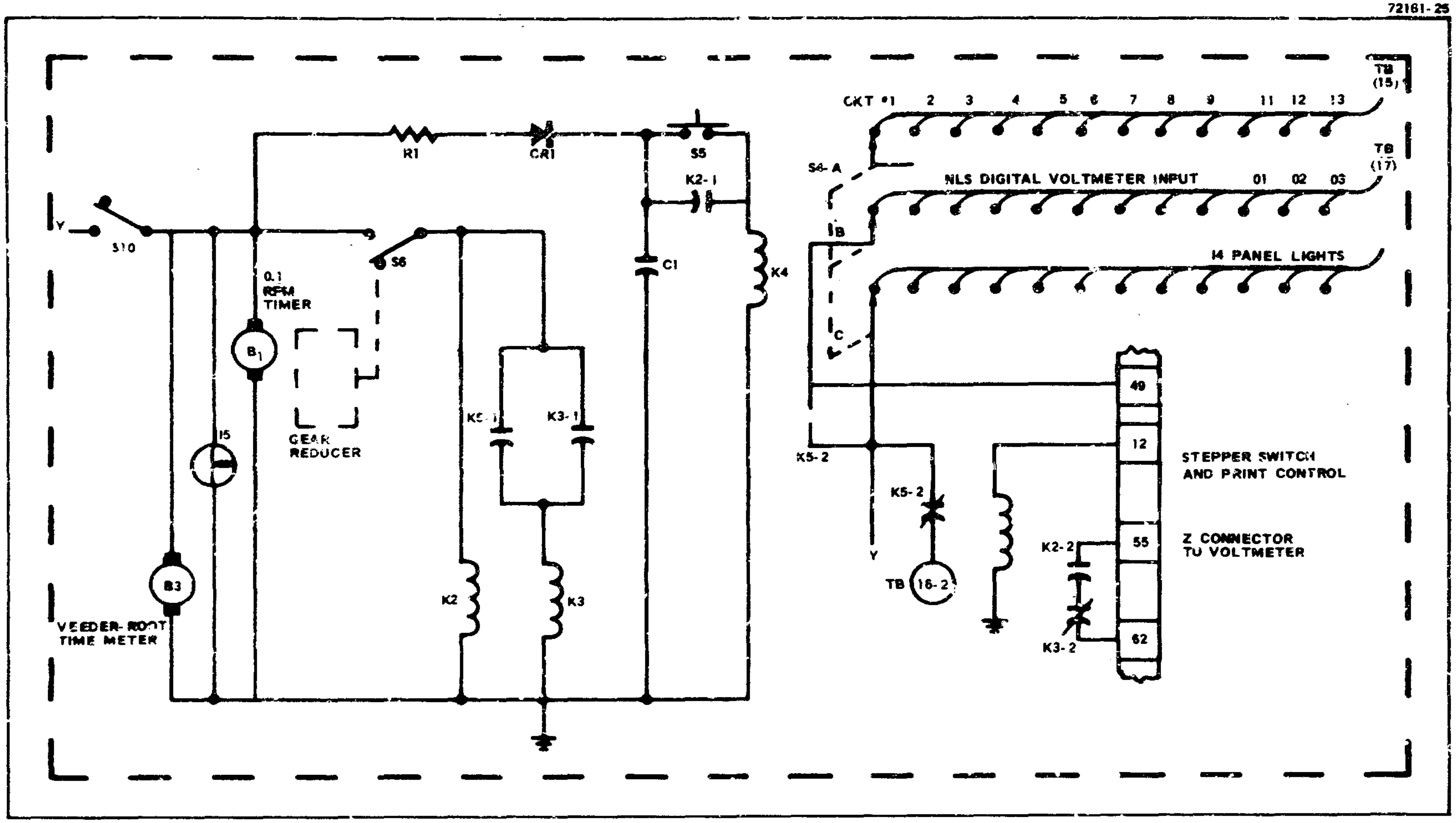

Figure 3.3.1a. Control Panel and PS Connection Diag.

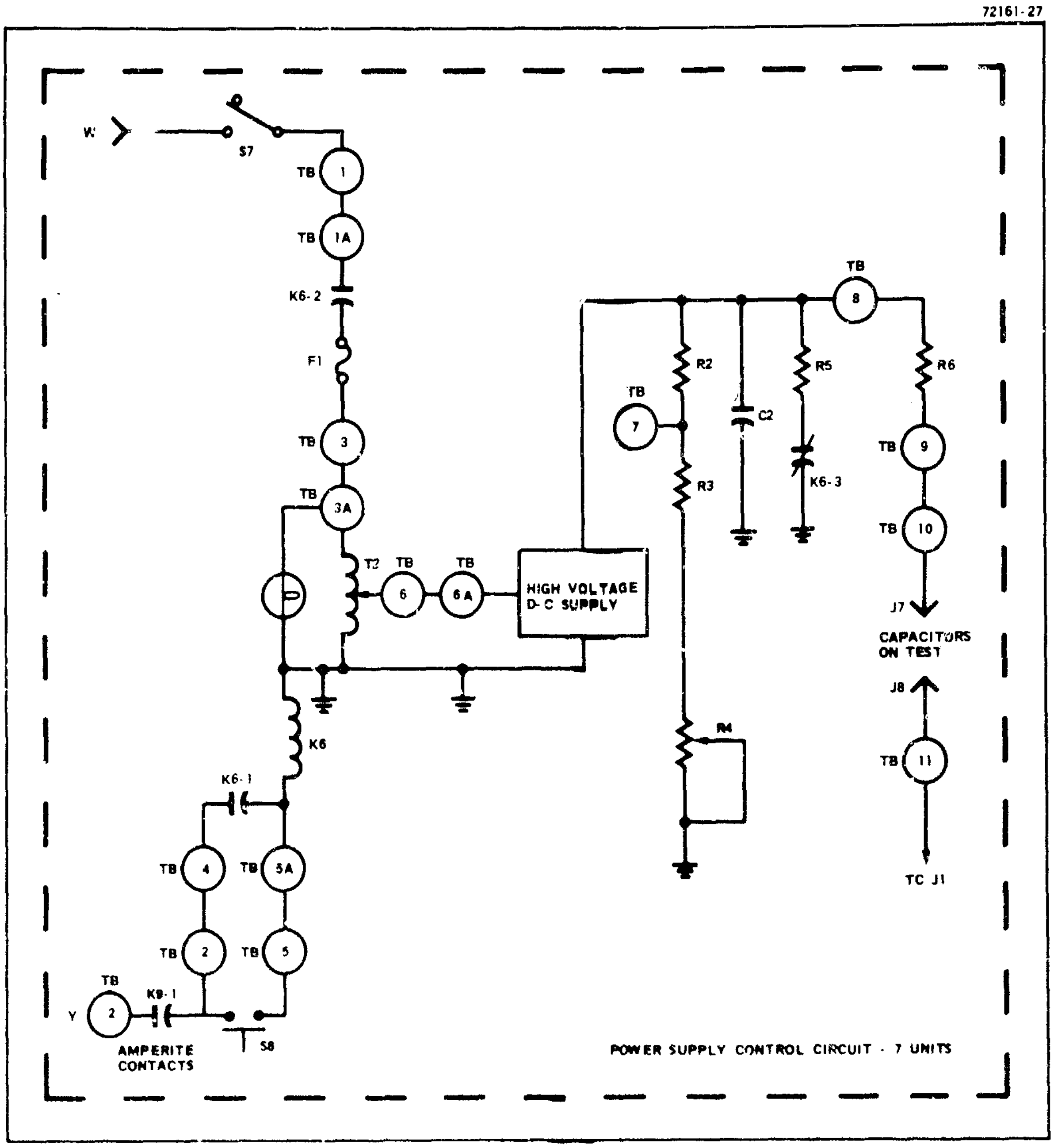

Figure 3.3.1b. Control Panel and PS Connection Diag.

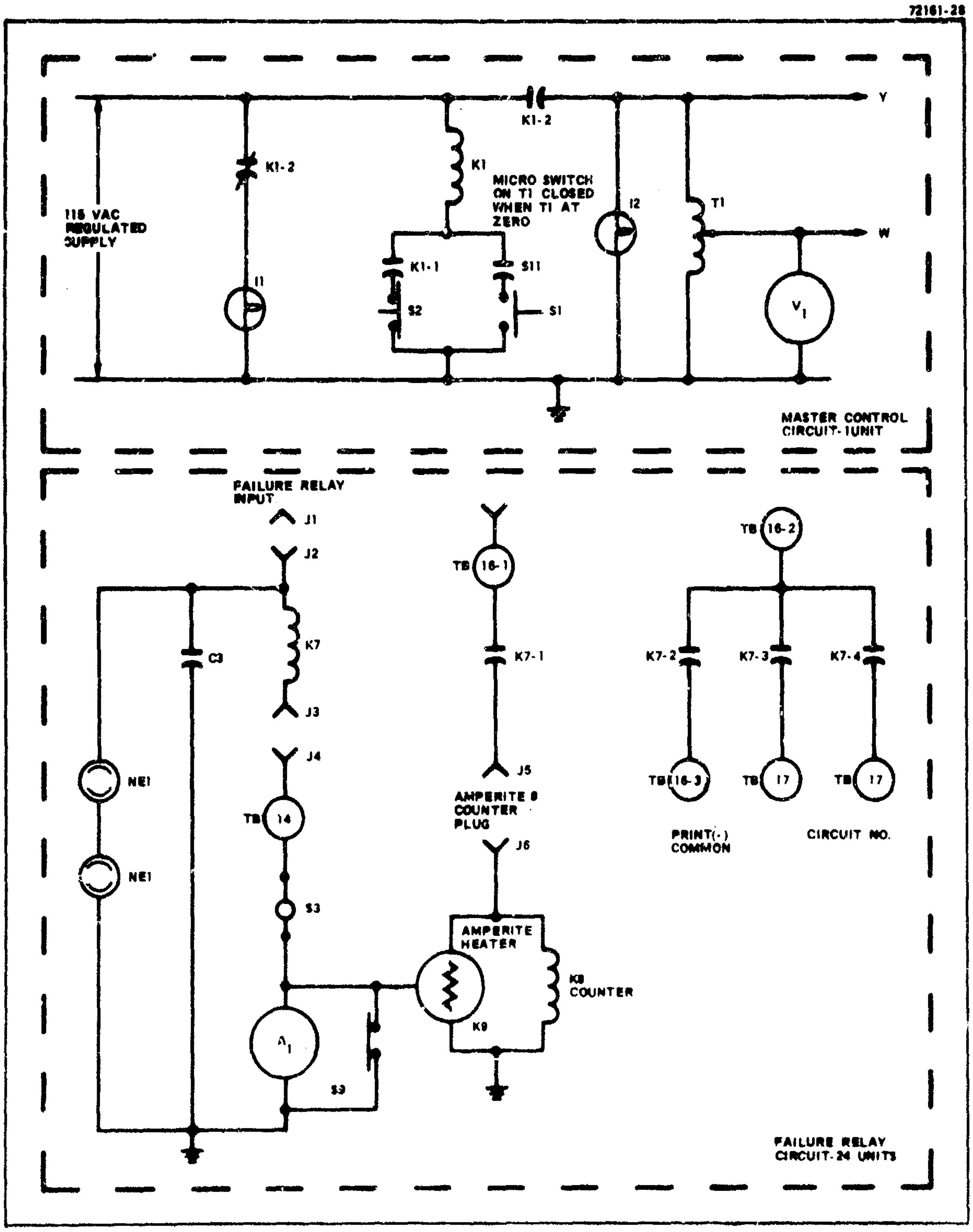

Figure 3.3.1c. Control Panel and PS Connection Diag.

3.4 PART NAME AND DESCRIPTION: Capacitors, Paper

No further description of the part is given.

ALT Evaluation	
Assumptions Generally Accepted	___
Empirical Proof	___
Algorithm	X
Physical Model	___

SOURCE: "Accelerated Life Testing on Paper Dielectric Capacitors" by Joseph Kimmel, 1958. (Reference 190.)

PURPOSE OF TEST: To investigate the progressively increasing stress method for use in the inverse power rule ALT model.

DEGREE OF VALIDATION

The results of this study indicate that either the progressive stress method for use in the inverse power model does not hold, or that the assumed values of k(5,6,7) were not the true values of the power exponent for this part, or that the failure times for this part are not distributed exponentially. The author further points out that there may be rate effects and other factors that influenced his results. The important thing in this author's presentation is that he developed a methodology for validation and attempted to prove the usefulness of the model. That his results did not conclusively validate the model is in itself an important finding.

DESCRIPTION OF TEST METHOD

Capacitors were selected from a lot and tested to failure in the equipment represented by the diagram in Figure 3.4.1. The accelerating stress used was d.c. voltage which was applied at a progressively increasing level. Two different uniform rates of voltage increase were applied to two samples of parts. The parts were tested to failure and the failure times were noted automatically with a chart recorder. A complete failure analysis was performed on all parts.

SUMMARY OF RESULTS

The objective of this study was to take two samples of parts from the same lot, subject them to progressively increasing voltages at two different rates, convert these failure times to equivalent failure times at constant stress, and determine if different rates of application of voltage would yield equal results for tests performed at constant stress levels.

The model under study is developed as follows:

1. Under the inverse power rule, voltage applied at a constant level yields the following estimate of failure time

$$L_2 = L_1\left(\frac{V_1}{V_2}\right)^k \qquad (3.4.1)$$

where L_2 = life at constant rated voltage V_2

L_1 = life at constant accelerated voltage V_1

k = power exponent

2. If a progressively increasing voltage is applied to a capacitor, it is theorized under the inverse power rule that formula 3.4.1 also applied but the voltage is expressed as

$$V_1 = ct \qquad (3.4.2)$$

where c = rate of change of voltage in volts/hour

t = failure time at progressively increasing voltage V_1

3. Equation 3.4.1 can then be expressed as

$$L_2 = \left(\frac{c}{V_2}\right)^k \frac{t^{k+1}}{k+1} \qquad (3.4.3)$$

where L_2 = estimate of equivalent life at constant stress level V_2

V_2 = any constant voltage including rated voltage

t = time to failure in progressive stress test

k = power exponent

4. Two samples of parts can then be tested to failure at two different rates of increase of progressive stress. The power exponent k is a constant for all voltages and is assumed a known value. The failure times from the progressive stress tests would be used in formula 3.4.3 to get several estimates of L_2 (equivalent life at rated voltage). The individual estimates of L_2 would be used to calculate a mean equivalent life for each rate of increase of voltage stress using the assumption that failure times are exponentially distributed. The mean of the L_2 values from each of the two rates should yield equal estimates if the model is valid.

This procedure was followed with 2 groups of parts at different c values. Seventeen parts were tested to failure at each stress level. The accelerated test failure times t were converted to estimates of L_2 using formula 3.4.3 and various assumed values of k (i.e., 5, 6, and 7). The results indicated that the 2 rates gave different estimates of the mean value of L_2 and hence it would appear that based on this evidence there

is little to prove the validity of the use of progressively increasing voltage as a method for implementing the inverse power rule model for this part.

INSTRUCTIONS FOR USE

1. Select a random sample of parts with the sample size large enough to yield statistically significant results (see Section 3.0).

2. Determine a rate of increase of voltage to be applied to the parts in order to reduce test time.

3. Test the parts to failure, note the failure times, and calculate an equivalent failure time at rated voltage with formula 3.4.3.
   Note: The value of k in formula 3.4.3 must be either known or assumed.

4. Calculate the estimate of mean life at rated constant voltage from the equivalent failure times of all the parts in the sample using the assumption that failure times are distributed exponentially.

LIMITATIONS/RANGE OF APPLICABILITY

The most important factor in the use of the subject ALT method is that the value of the k (the power exponent) in the inverse power rule must be known. A small error in its estimation will result in a large error in the calculation of the estimate of mean life at rated constant stress. The method has been attempted on several varieties of capacitors with varying degrees of success.

REFERENCES: 190

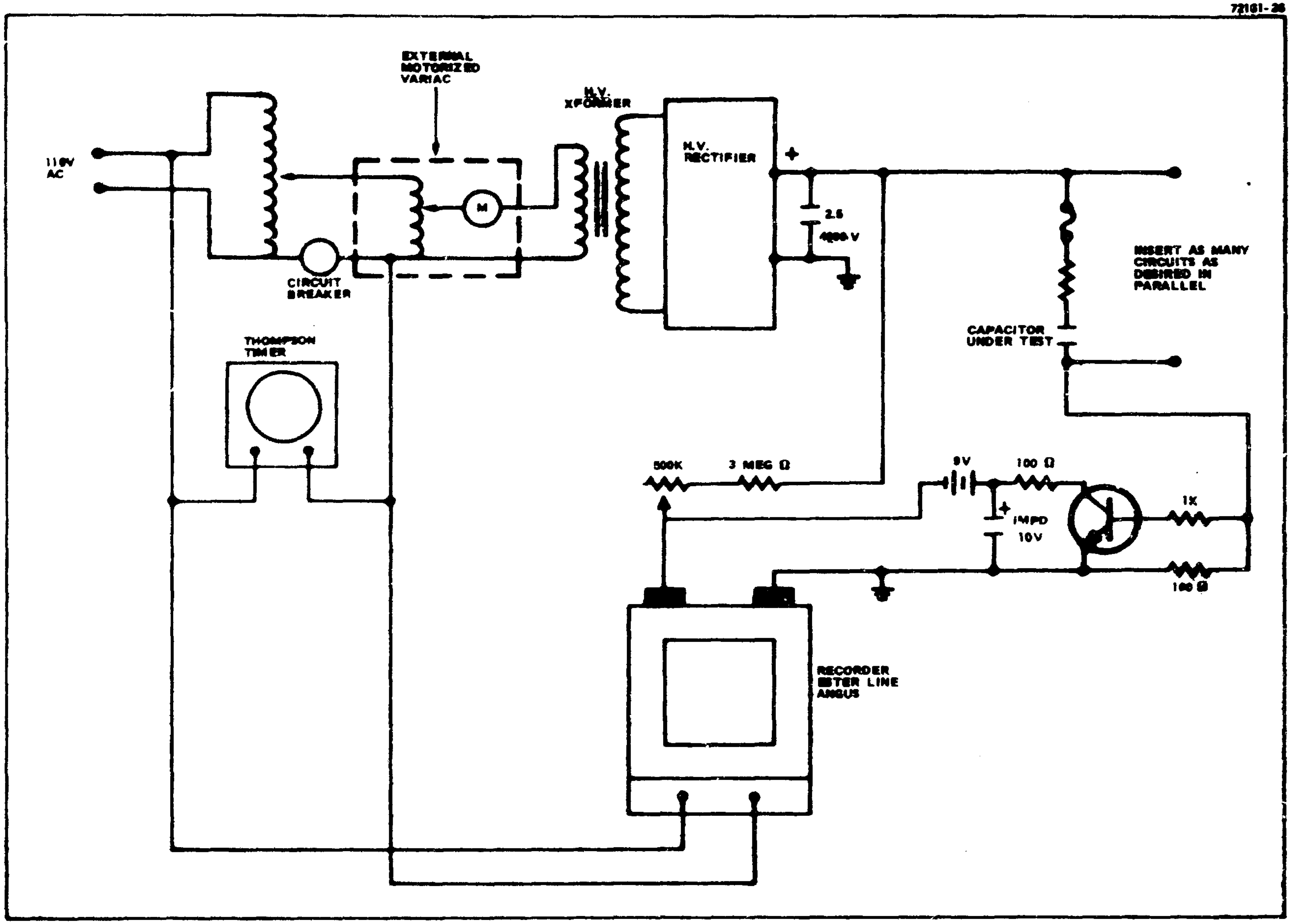

Figure 3.4.1. Stress Test Circuit

3.5 PART NAME AND DESCRIPTION: Capacitors, Mylar

ALT Evaluation	
Assumptions Generally Accepted	___
Empirical Proof	___
Algorithm	X
Physical Model	___

No detailed description of the parts tested was presented.

SOURCE: The results were presented in a 1957 paper in the IRE Transactions by G. J. Levenbach. (Reference 216.)

PURPOSE OF TEST: To determine the mean life of these parts based on accelerated life tests using voltage as the acceleration stress and the inverse power rule for converting the results to equivalent life at rated voltage.

DEGREE OF VALIDATION

This section of the handbook simply demonstrates the use of the inverse power rule for mylar capacitors without claiming validation. It was a small experiment on a new product but serves its purpose of calculating an estimate of p. The user of the method is instructed in the methodology but will have to generate his own results before employing the method.

DESCRIPTION OF TEST METHOD

The experimental design consisted of testing to failure 20 parts at each of 11 different combinations of three stresses. Two levels of temperature, 4 levels of voltage, and 3 levels of burn in test were included in the test. No numerical values were given for the stresses, hence no actual relationships can be determined from this report except to demonstrate the use of the inverse power rule. Not all of the cells of the experimental design were attempted and the cell at rated voltage and temperature was one which was omitted.

SUMMARY OF RESULTS

The model used for converting accelerated test results to estimates at rated (or less severe) stress levels is the inverse power rule which is given as:

$$\frac{\theta_1}{\theta_2} = \left(\frac{V_2}{V_1}\right)^p$$

where θ_1 = mean life at rated voltage (or at some voltage $< V_2$)

θ_2 = mean life at accelerated voltage

V_2 = voltage at accelerated level

V_1 = voltage at rated level (or some level < V_2)

p = power exponent

Since in this test program estimates of θ_1 and θ_2 were available based on empirical results, the major problem was the determination of p. This is necessary for use in future accelerated tests of this part type when it will only be necessary to test a sample of parts at V_2. The power exponent p is estimated by plotting the voltage versus the mean failure times on log-log paper. p is the negative slope of the line through the observed data points. The 90% confidence limit estimate of p from the data is given as between 4 and 6.5 for mylar capacitors.

INSTRUCTIONS FOR USE

The following steps should be employed for the use of this ALT method:

1. Test a large enough sample of mylar capacitors to failure to provide statistically significant results at rated and at several levels of accelerated voltage (see Section 3.0).

2. Calculate the MTBF for each voltage level assuming an exponential failure distribution.

3. Plot the logarithm of the MTBF's versus the logarithm of voltages. The negative slope of the line of best fit through the data points represents an estimate of p the power exponent.

4. Calculate confidence limits on p using the chi square with 2 r degrees of freedom.

5. Perform all future tests on these parts at accelerated voltage only. Estimate life at rated stress using the observed value of θ_2, V_1, V_2, and the value of p calculated in Steps 1-3.

LIMITATIONS/RANGE OF APPLICABILITY

One of the limitations of this method is that the accurate estimation of p is a very important factor. A small error in this exponent results in a large error in the estimate of θ_1.

The exponential assumption for failure times has been assumed in this case whereas many pieces of literature indicate otherwise for capacitors.

Many authors have, however, reported reasonably successfully on the use of this method for nearly all types of dielectrics. Its merits in a given application should be conclusively proven prior to use.

REFERENCES: 30, 55, 109 and 216.

3.6 PART NAME AND DESCRIPTION: Capacitors, Paper with Chlorinated Impregnants.

ALT Evaluation	
Assumptions Generally Accepted	
Empirical Proof	
Algorithm	X
Physical Model	

The results obtained on this ALT method were from experimental capacitors made with three sheets of .4 mil kraft paper and aluminum foil. The rated voltage was 600 volts DC and the capacitance was .25 microfarad.

SOURCE: The test results for these capacitors were in a paper in the 1944 AIEE Transactions by L. J. Berberich, C. V. Fields, and R. E. Marbury. (Reference 30.) The date of the paper attests to the long use of the subject method.

PURPOSE OF TEST: To estimate mean life of capacitors at rated voltages based on results at overstress voltages.

DEGREE OF VALIDATION

This ALT method has been used by many people on many parts with varying degrees of success. The major problem with the method is that n, the power exponent must be accurately found (or known) in order to get reasonably accurate estimates. It appears that only two data points were utilized in the solution of the estimate of n hence the values presented have not been proven.

DESCRIPTION OF TEST METHOD

The tests carried out were preceded by a complete series of probing tests directed at maintaining the same failure mode at the various stress levels. Tests were run in the range from 500-1500 volts and 70-100°C.

SUMMARY OF RESULTS

This method is exactly the same as that reported in Section 3.5. It is based on the relationship

$$L_1 = L_2 \left(\frac{E_2}{E_1}\right)^n$$

where L_1 = mean life at the lower voltage E_1

L_2 = mean life at the higher voltage E_2

n = power exponent.

The tests conducted were directed toward the discovery of the n value for parts treated with two different impregnants. For capacitors treated with Conventional Inerteen the average value of n was 3.2. For parts

treated with Conventional Inerteen plus 0.1% of Stabilizer 1 the estimated average n was 3.7.

INSTRUCTIONS FOR USE

1. Test to failure a large enough sample to give statistically significant results at both rated and at several overstress voltages (see Section 3.0). Calculate the mean life at each stress level.

2. From the formula

$$L_1 = L_2 \left(\frac{E_2}{E_1}\right)^n \text{ solve for } n$$

3. Perform all future tests at voltage E_2 and solve the above equation for L_1 using the n calculated in Step 2.

LIMITATIONS/RANGE OF APPLICABILITY

Same as in Section 3.5.

REFERENCES

30, 55, 109 and 216.

3.7 PART NAME AND DESCRIPTION: Capacitor, Solid Tantalum.

This ALT method was developed for use on all types of Kemet solid tantalum capacitors of the type covered by Mil-C-26655.

ALT Evaluation	
Assumptions Generally Accepted	___
Empirical Proof	___
Algorithm	X
Physical Model	___

SOURCE: This accelerated life testing method was reported in several publications during the period ranging from 1961-64 by G. H. Didinger, Jr. of the Kemet Division of Union Carbide. (References 90, 91 and 92.)

PURPOSE OF TEST

The primary purpose of the test method is to estimate the failure rate of the parts on hand for use in applications with a given mission time and a specified reliability requirement. The method can also be used for estimation of the reliability of parts at rated voltages and temperatures derived from accelerated tests at higher than rated voltages and temperatures. Tests have indicated that failure times are distributed according to the Weibull distribution with a decreasing failure rate (i.e.: the Weibull shape parameter is less than 1). Furthermore, the Weibull shape parameter is reported in these papers as remaining constant over the range of overstress voltages and temperatures for which acceleration factors have been calculated.

DEGREE OF VALIDATION

This method has not been validated in the literature that has been found. Reference 90 of this section contains the statement that the Weibull shape parameter (β) is the same at both accelerated and at normal stress levels. This condition must exist in order for the model and its attendant acceleration factors to apply. No evidence is presented which substantiates the claim of constant shape factor for these parts. In fact, Reference 92 at the end of this report gives some Kemet data which implies that β does change at different stress levels. Also, the records which accompanied incoming lots of these part types have been inspected with the result that the estimates of the Weibull shape parameter do in fact vary widely at differing stress levels.

DESCRIPTION OF TEST METHOD

A schematic diagram is presented in Figure 3.7.1 to describe the accelerated test method utilized. It was pointed out in the literature that no protective resistance in series was used with the parts on test for the purpose of encouraging catastrophic failures. The only failure mode observed in these tests was the short circuit. The definition of a catastrophic short circuit failure was when the leakage resistance of a part was equal to or greater than its capacitive reaction at 120 cycles/second.

The environmental stress applied to the capacitors on test was temperature at the 85 and 125°C levels. Seven levels of voltage were used in the development of acceleration factors. Sample sizes varied from 10 to 27 at each stress level.

SUMMARY OF RESULTS

The failure times of the parts tested were distributed according to the Weibull distribution. The failure times were plotted on Weibull probability paper with failure time on the abscissa and either the median rank or $\frac{i}{n+1}$ on the ordinate. The Weibull shape and scale parameters are estimated from plotting the failure times, drawing a line of best fit through the points and proceeding as specified on Weibull probability paper.

The acceleration factors which have been developed are as follows:

Acceleration Factor	Ratio of Test Voltage to Nameplate Voltage 85°C	125°C
1	1	.67
10	1.16	.81
100	1.32	.96
1,000	1.47	1.12
10,000	1.63	1.28
100,000	1.79	1.44
1,000,000	1.94	1.60

The algorithm for calculation of the acceleration factors is

$$A = \left(\frac{\alpha_N}{\alpha_A}\right)^{1/\beta} \tag{3.7.1}$$

where: A = the acceleration factor corresponding to the stress condition.

α_A = Weibull scale parameter at accelerated stress levels.

α_N = Weibull scale parameter at rated stress levels.

β = Weibull shape parameter for both accelerated and rated stress levels.

The following example demonstrates the manner in which the accelerated test method can be used in actual practice:

1. Given the following accelerated test conditions:

   Sample Size = 1,000
   Test Temperature = 85°C
   Test Voltage = 1.63 X Nameplate Voltage
   Acceleration Factor = 10,000

2. Perform the accelerated life test until 1% of the sample has failed. Align the failure times with their respective mean ranks as follows:

Failure #	Cumulative % Failed	Failure Time (Hours)
1	.1	.255
2	.2	1.0
3	.3	2.25
4	.4	4.0
5	.5	6.4
6	.6	9.3
7	.7	12.8
8	.8	16.9
9	.9	21.0
10	1.0	26.0

   Note: Author used $\frac{i}{n}$ instead of median rank or $\frac{i}{n+1}$ in Cumulative % Failed.

3. Plot the data on Weibull probability paper and draw the line of best fit through the points. From this operation, it is possible to estimate β, the Weibull shape parameter and α, the Weibull scale parameter. In this example $\hat{\beta}_A = .5$ and $\hat{\alpha}_A = 492.75$.

4. The next step is to substitute these values into the algorithm and solve for $\hat{\alpha}_N$, an estimate of the Weibull scale parameter had these parts been run at nameplate voltage. This gives the following result:

$$\alpha_N = \alpha_A x A^{\beta} \tag{3.7.2}$$

$$\alpha_N = 492.75(10,000)^{.5}$$

$$\alpha_N = 492.75(316.23)$$

$$\alpha_N = 155,822$$

5. This value of α coupled with the assumed constant value of β of .5 can then be used to calculate the reliability at any given number of cycles with the following formula:

$$F(x) = e^{-\frac{x^{\beta}}{\alpha}} \qquad (3.7.3)$$

$$F(10^6) = e^{-\frac{(10^6)^{.5}}{155,822}}$$

$$F(10^6) = e^{-\frac{1000}{155,822}}$$

$$F(10^6) = e^{-.00643}$$

$$F(10^6) = .994$$

Therefore, the probability that these parts would last one million cycles when operated at rated voltage and a temperature of 85°C is .994.

INSTRUCTIONS FOR USE

1. Construct test equipment as shown in Figure 3.7.1.
2. Test parts until a reasonably large number of failures are observed.
3. Plot the failure times on Weibull probability paper and draw the line of best fit through the data points.
4. Estimate $\hat{\alpha}_A$ and $\hat{\beta}_A$ from the Weibull plot.
5. Use the algorithm given previously for calculating an estimate of α_N.
6. Using α_N and β_N calculate an estimate of the reliability, characteristic life, mean life or any other parameter of interest. These estimates will be of what would have been found had the parts been tested at rated stress levels.

LIMITATIONS

The major assumption upon which this method is based is that the Weibull shape parameter (β) is equal at both rated and at overstress levels. Therefore, this method should be used with care unless it can be conclusively proved that this in fact is the case for this part type.

REFERENCES

90, 91, 92 and 233.

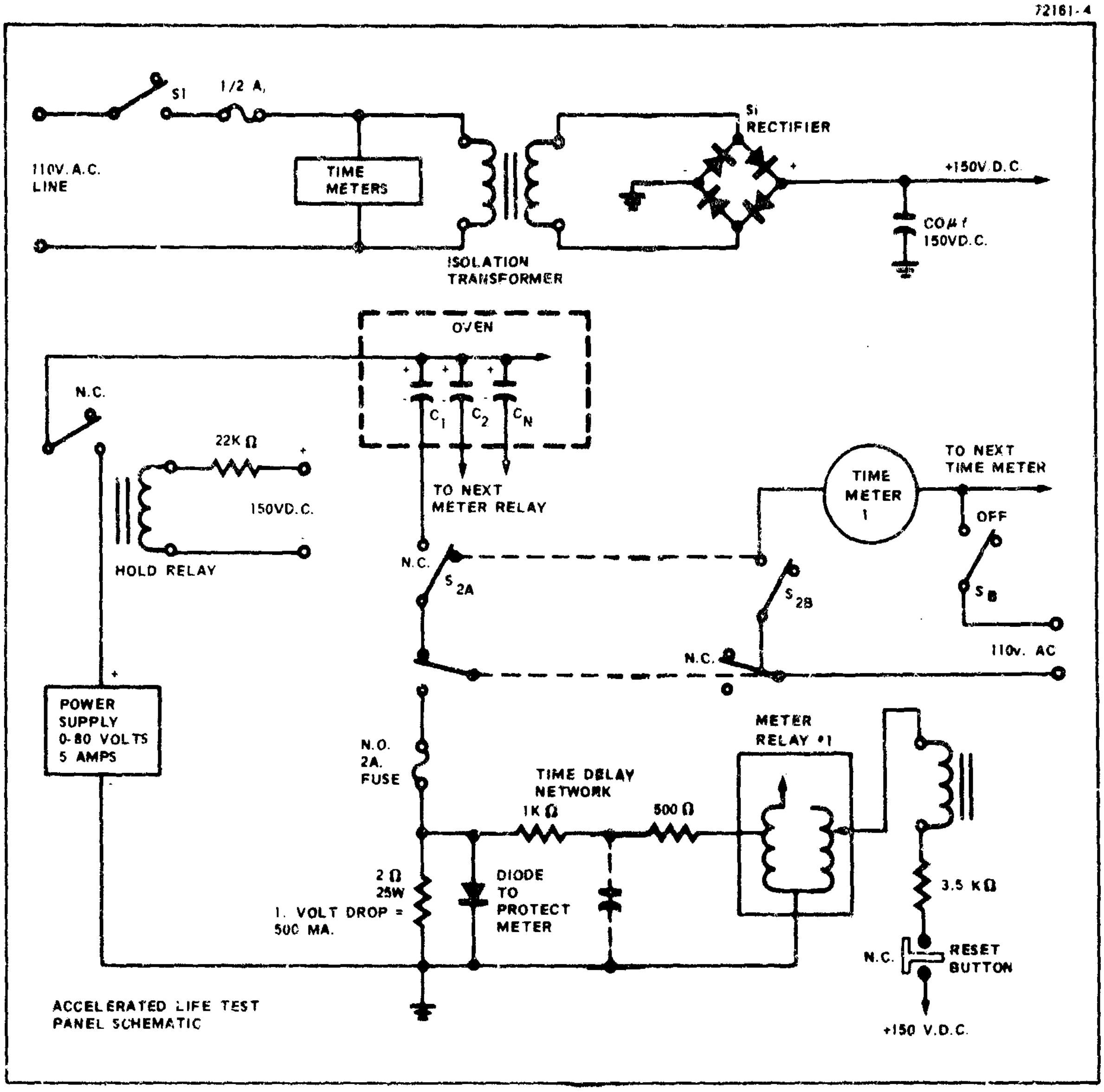

Figure 3.7.1. Accelerated Life Test Panel Schematic

3.8 PART NAME AND DESCRIPTION: Heating Element, Calrod

ALT Evaluation	
Assumptions Generally Accepted	X
Empirical Proof	X
Algorithm	X
Physical Model	

Model P7 self-cleaning oven, electric range.
Manufacturer - General Electric Co., Range Department, Louisville, Ky.
Design Power - 3,000 watts.
Design Voltage - 236 volts.
Application - home kitchens.
Average Customer Use - 1,250 energized hours over 20 years.
Product Use Specification - 3,000 energized hours over 20 years to satisfy 90% of the users.

SOURCE: "The Use of Accelerated Testing in Reliability Assessment" by Albert Osse, Range Department, General Electric Co., Louisville, Ky. (Reference 261.)

PURPOSE OF TEST: To estimate the mean life of production Calrod units at normal operating conditions.

DEGREE OF VALIDATION

This method is not validated, because the author did not include the following data:

1. The sizes of the groups within the sample tested at the different voltage levels of 248, 260, 280, 300, 320, and 340 volts, and how these sizes were determined.

2. None of the test data was presented (e.g., verification of normality at each voltage level).

3. The model (Equation 3.8.1) graphs were not presented.

DESCRIPTION OF TEST METHOD

1. Definition of Failure:

   Cessation of life by "open" in resistance element. Failure is a function of temperature, which is directly related to applied voltage.

2. Test Equipment:

   Test Rig - Simulated oven.

   Test Set-ups - 144 simulated ovens in temperature controlled room, most with 248 v supplies, but some 260, 280, 300, 320, and 340 v.

3. Method of Gathering Data:

Monitor continuously during the test on each unit: the Calrod unit sheath temperature, cold resistance, wattage, leakage current, etc.

4. Method of Applying Accelerating Stress:

Applied Accelerating Stress - voltage.
(Note: Design voltage is 236, but home voltage is approximately normally distributed about 240 volts as a mean. To include the upper 90% level of this distribution the normal test voltage must be 248 volts. 248 volts is also NEMA standard.)

Duty Cycle - NEMA standards specify 60 minutes ON followed by 20 minutes OFF.

Sample Size - 100 units, a continuous run from one day's production.

Procedure - Divide units and run to failure at various voltage levels in accordance with ON-OFF duty cycle.

SUMMARY OF RESULTS

1. Mathematical Models

Definitions:

V_o = Normal test voltage

V_i = Elevated test voltage

H_o = Mean life (hours) at V_o

H_i = Mean life (hours) at V_i

S_o = Standard deviation at V_o

S_i = Standard deviation at V_i

$H_{o,LL}$ = Lower limit of mean life at V_o

$H_{i,LL}$ = Lower limit of mean life at V_i

$S_{o,LL}$ = Upper limit of standard deviation at V_o

$S_{i,UL}$ = Upper limit of standard deviation at V_i

$H_{o,UL}$ = Upper limit of mean life at V_o

$H_{i,UL}$ = Upper limit of mean life at V_i

The relationship between the mean lives at normal test voltage (V_o) and the elevated voltage (V_i) can be expressed by the following Model:

$$\frac{\bar{H}_o}{\bar{H}_i} = K\left(\frac{V_i}{V_o}\right)^{\alpha} \qquad (3.8.1)$$

Evidently (since H, not $\bar{H}$, has been defined) the author intends H to be a simple mean.

2. Analysis Methods

a. The distribution of failure times for units was normal at each of the 248, 260, 280, and 300 volt levels, as verified by both graphic presentation on normal probability paper and by the Kolmogrov-Smirnov "Goodness of Fit" test. Mean life (hours) with respect to test voltage is shown graphically in Figure 3.8.1.

b. The results above the 300 volt level were not reliable, because the failure mode was not the same as that for the lower voltage levels. Therefore, results above the 300 volt level were discounted in the mathematical model.

c. By utilizing the mathematical model, the data generated by the test, and a plot on log-log paper of the ratios of $\bar{H}_o/\bar{H}_i$ versus V_i/V_o, it is possible to estimate the values of K and α.

d. With values of K and α established, tests can be run on this particular type Calrod unit at some elevated voltage V_i (not greater than 300 volts), and the estimated value of the mean life can be computed for a lower test voltage. (It can also read from the 50% curve in Figure 3.8.1.)

e. The upper and lower limits of the mean life at normal voltage (V_o) at a specified confidence level can be calculated as shown below.

At any one voltage level the distribution of failure hours is normal; therefore, the limits of the mean can be estimated from a table of normal standard deviates, using a "t" distribution. (This is defined as the [absolute value] difference between the mean of a sample and the true mean of the population from which the sample was drawn, divided by the estimated standard deviation of the mean.)

The estimated lower limit of mean life at elevated voltage V_i is:

$$\bar{H}_{i,LL} = \bar{H}_i - t\,(S_i/\sqrt{N})$$

The estimated upper limit of mean life at elevated voltage V_i is:

$$\bar{H}_{i,UL} = \bar{H}_i + t\,(S_i/\sqrt{N}$$

The lower and upper limits of mean life at normal voltage V_o is found by using the model:

$$\bar{H}_{o,LL} = \bar{H}_{i,LL}\left[K\left(\frac{V_i}{V_o}\right)^{\alpha}\right] \quad \text{for lower limit}$$

$$\bar{H}_{o,UL} = \bar{H}_{i,UL}\left[K\left(\frac{V_i}{V_o}\right)^{\alpha}\right] \quad \text{for upper limit.}$$

f. The expected percent of the units that will fail to meet the established life requirement (3,000 hours)* at normal voltage (248 v) can be calculated in the following manner:
*(Or other established life requirement)

Calculate the estimated standard deviation S_o at V_o:

$$S_o = S_i\left[C\left(\frac{V_i}{V_o}\right)^{b}\right] \quad \text{(same form as the model)}$$

The constants C and b are established in the same manner as K and α.

The standard deviate Z is:

$$Z = \left[\frac{3{,}000 - \bar{H}_o}{S_o}\right].$$

Reference to a Table of Probabilities in the Normal Distribution will give the percent of the units that will fail to meet this criterion of 3,000 hours.

INSTRUCTIONS FOR USE

This ALT is a useful tool for rapidly checking production line Calrod units, suspected field problems, or other similar problems where the time factor is critical for problem solution.

The following sample problem and its step-by-step solution will serve to illustrate how this ALT may be employed:

Problem: Suppose reports are received from field service engineers that comparatively large numbers of Calrod unit failures ("opens" in the resistance elements) are being experienced in home electric ranges.

1. The Reliability Engineering Department requests that a small number (3 or 4) of good Calrod units be shipped in from the field by the Service Department for examination.

2. The units are first completely inspected to verify conformance to drawings and specifications.

3. Next, the units are installed in the laboratory simulated ovens for test and subjected to an accelerated stress of 300 volts until failure.

4. The results may be checked in two ways: by means of the curves in Figure 3.8.1 or mathematically.

   a. By means of Figure 3.8.1:

      Plot the units recorded failure times at the 300 volt (V_i) stress level.

   b. By mathematics:

      In the model $\frac{H_o}{H_i} = K\left(\frac{V_i}{V_o}\right)^{\alpha}$

      The values of K and α were determined, approximately, by means of Figure 3.8.1 and plotting the results on log-log paper. As a check, K and α were also calculated by a computer program, using the "method of least squares." K and α were found to approach 1.318 and 9.888 respectively.

      Therefore, since the constants are now known, the equation can be solved for any values of H_i recorded in the tests. For instance, if one unit failed at 600 hours at 300 volts:

$$H_o = H_i \quad K\left(\frac{V_i}{V_o}\right)^{\alpha}$$

$$H_o = 600 \times 1.318 \left(\frac{300}{248}\right)^{9.888}$$

$$H_o \cong 4{,}820 \text{ hours}$$

LIMITATIONS/RANGE OF APPLICABILITY

This ALT is applicable only to the specific type of Calrod unit tested. The same logic will apply to units of other designs, but the mode or mathematical distribution of failure must be confirmed for each design

configuration or characteristics, and the formula constants established for each case. In addition, the ALT is only applicable to test voltage levels not exceeding 300 volts.

REFERENCES

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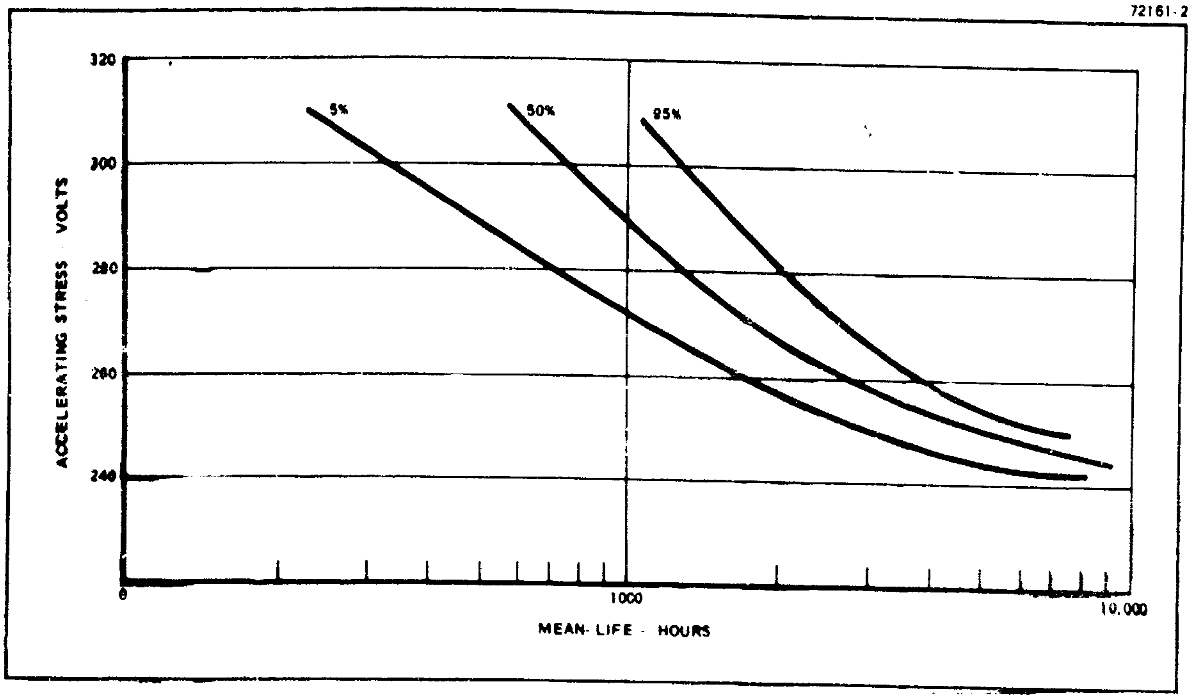

Figure 3.8.1. Voltage-Life Relationship

3.9 PART NAME AND DESCRIPTION: Insulation Materials

ALT Evaluation	
Assumptions Generally Accepted	___
Empirical Proof	___
Algorithm	X
Physical Model	___

The tests reported here are on specimens of two types of dielectric insulation material. They were constructed in the hope of simulating the failure mechanisms observed in high voltage generator insulation which is subjected to overvoltages for long periods of time. The two types of specimens used were made of Mylar polyester film and an asphalt-mica composite.

All the Mylar film was obtained from the same roll. It was .002 inches in thickness and a five layer pad was prepared for the test. The asphalt-mica specimens were constructed very elaborately with a uniform fault built into each.

SOURCE: The materials of interest were tested using overstress voltages to evaluate the inverse power rule model. The results are reported in Reference 112 titled "Progressive Stress - A New Accelerated Approach to Voltage Endurance", by W. T. Starr and H. S. Endicott in 1961.

PURPOSE OF TEST: To calculate the mean life of ce. .in dielectric insulation materials at rated voltages based on tests performed at overstress voltages.

DEGREE OF VALIDATION

The results in the use of the inverse power model as an accelerated life test method for Mylar and mica-asphalt dielectric specimens indicate that a log-log relationship between voltage and failure time may exist over portions of the stress-time spectrum. It is impossible to tell from the data presented whether or not the same results could be expected in subsequent repetitions of the tests at either accelerated or at rated stress levels. Additional verification tests are required to evaluate the applicability of the inverse power rule to specimens of these types.

DESCRIPTION OF TEST METHOD

A stabilized 60 cps voltage of 115 $\pm$ .3 V was applied through a motor driven variac and transformer to specimens about .010 inches thick. The voltage was applied until failure occurred. The specimens were placed between two electrodes of no. 304 stainless steel. The high voltage electrode was 1 inch in diameter, $\frac{1}{2}$ inch high with 1/8 inch radius, and with the active surface polished to a mirror finish. The ground electrode was the same except it was 2 inches in diameter. The tests were conducted at 23.6 $\pm$ 1.1°C and at 50 $\pm$ 2% relative humidity. No information is presented regarding sample size. It was not possible to accurately determine the various voltage stresses used in the test program.

SUMMARY OF RESULTS

The methods for calculating n from both constant stress and progressive stress tests or from combinations of them are those described in subsections 3.2, 3.3, and 3.4 of this handbook. The major finding from the results of the tests was that apparently a change in failure mode occurred at about 100 hours causing a change in the slope of life versus voltage curve for the Mylar specimens.

For Mylar polyester film specimens, the calculated value of n was 4.8 for test times less than 100 hours. For test times greater than 100 hours (i.e., lower voltage stresses) the calculated value of n is 1.44. For the mica asphalt specimens the break in the life versus voltage line occurred at about 10 hours. The calculated n prior to 10 hours was 18.4 while for specimens that lasted longer than 10 hours (presumably because they were tested at lower voltage stresses) the n was computed to be 11.2.

INSTRUCTIONS FOR USE

1. Test a sample of specimens to failure using either constant voltage, progressive voltage, or a combination of them as accelerating stresses (see Section 3.0).

2. Plot the failure times versus voltage on log-log paper.

3. Draw a straight line through the plotted points (if possible). If more than one straight line fits different portions of the data, this information can be used in the accompanying search for different failure modes.

4. Calculate n for each range of stresses. The slope of the line fitting the data will be $(-\frac{1}{n})$.

5. Use the calculated value of n in future accelerated tests in the inverse power rule formula to estimate mean life of these specimens at rated voltage stress.

LIMITATIONS/RANGE OF APPLICABILITY

Since no information was presented regarding sample size it was difficult to evaluate the significance of the information gathered. It does seem though that in designing an ALT method one should attempt to use the stress ranges developed by the authors that will yield only a single failure mode (although this is not a complete necessity).

The use of the inverse power rule model has been attempted on many dielectric materials. However, the specific results obtained very likely apply only to those specimen types studied in Reference 112.

REFERENCES: 111, 112, 113, 190.

3.10 PART NAME AND DESCRIPTION: Relay, Crystal Can

ALT Evaluation	
Assumptions Generally Accepted	X
Empirical Proof	X
Algorithm	X
Physical Model	

These parts are of the type MIL R 5757/10 RY4NA3B3L01 except for a contact rating of 3 amps and 3 inch leads. The parts were produced by four different manufacturers. They are double throw double pole relays with a contact voltage of 28 volts DC, actuation rate to 30 cycles/ second, coil resistance of 675 ohms, coil voltage 26.5 volts, and are designed to operate in a temperature range from -65°C to +125°C. The identity of the parts manufacturers is not given in the referenced reports.

SOURCE: This ALT method was developed and validated in two study contracts for RADC by Hughes Aircraft Company, Fullerton, California in 1964 and 1965. Details of the studies are in References 297 and 298.

PURPOSE OF TEST: To develop an ALT method based on time transformations of the distribution functions of parts tested to first miss failure which will yield quantitative estimates of reliability characteristics at rated stresses.

DEGREE OF VALIDATION

Two successive series of tests were performed on the exact same part type using substantially the same test method which yielded comparable results. The two test programs were conducted approximately a year apart with parts selected from the same manufacturer's production lots. The main drawback to more complete validation of the test method is that small sample sizes were used in both test series. No physical model has been found to satisfy the empirical results.

DESCRIPTION OF TEST METHOD

The relays were tested on specially designed equipment which is described in block diagram form in Figure 3.10.1. The test equipment had provisions for actuating the parts at several contact loads, actuation rates, and ambient temperatures. It detected first, second, and third miss failures on each of ten test parts and recorded the number of cycles at each failure by actuating a 35 mm. camera which photographed a display board each time a failure occurred.

The duty cycle of the parts was 50% on. The electrical load was connected in series with the normally opened and the moving contact of each relay. The load for each relay contact pair consisted of a fixed noninductive wire wound resistor in series with a variable wire wound vernier resistor. A square wave generator was utilized to drive a mercury wetted contact chopper type relay at the desired actuation rate. A precision laboratory oven was used to control the ambient temperature. Various operating parameters of the relays were measured prior to the start of the tests and periodically throughout the life of the parts. Measurements were recorded on contact resistance, contact bounce, coil resistance, operate time, release time, pickup voltage, and dropout voltage.

SUMMARY OF RESULTS

An analysis of variance was performed on the number of cycles to first miss failure and to third miss failure to identify those stresses and interactions of stresses which contributed to the reduction of relay life times. Tests were run at normal stress levels and at 26 other combinations of stresses which constituted accelerated stresses. The analysis indicated generally that relays produced by different manufacturers have different life characteristics and that the application of contact current, temperature and actuation rate in combination significantly reduce relay life and hence are useful accelerating stresses.

The underlying distribution of failure times is Weibull. The predominant failure made was material transfer between contacts.

A numerical example will illustrate the procedure. In RADC TR 65-46 five relays from Manufacturer B were tested at 3 amperes contact load, one cycle/second actuation rate, and 25°C ambient temperature. The cycles to failure for these parts were plotted on Weibull probability paper and the shape parameter (β) was estimated to be 1.68 and the scale parameter (α) was estimated as 8.15×10^9. This test run was defined as manufacturers rated stress. In the same study five relays were tested to failure at a combined overstress condition consisting of a contact current of 6 amps, actuation rate of 30 cycles/second and 25°C ambient temperature. Again the failure times were distributed according to the Weibull with the shape parameter estimated as 1.11 and the scale parameter estimated as 1.409×10^6. In a subsequent study reported in RADC TR 66-425, 10 relays were tested at both rated and accelerated conditions as defined above. The subsequent tests at rated stresses yielded estimates of Weibull parameters as follows: $\beta = 2.73$ and $\alpha = 1.73 \times 10^{17}$. The tests at accelerated stresses yielded $\beta = 1.13$ and $\alpha = 3.16 \times 10^6$. These data were inserted in the following algorithm:

$$\tilde{\alpha}_N^* = \left(\frac{\tilde{\alpha}_N \tilde{\beta}_A}{\tilde{\alpha}_A \tilde{\beta}_N}\right) \frac{\tilde{\alpha}_A^* \tilde{\beta}_N^*}{\tilde{\beta}_A^*} \qquad \tilde{\beta}_N^* = \tilde{\beta}_A^* + (\tilde{\beta}_N - \tilde{\beta}_A)$$

where:

$\tilde{\alpha}_N^*$ = estimate of Weibull scale parameter for parts if they had been tested at manufacturer's rated operating and environmental stresses

$\tilde{\beta}_N^*$ = estimate of Weibull shape parameter for parts if they had been tested at manufacturer's rated operating and environmental stresses

$\tilde{\alpha}_A^*$ = estimate of Weibull scale parameter obtained from a current test run at accelerated stresses

$\tilde{\beta}_A^*$ = estimate of Weibull shape parameter obtained from a current test run at accelerated stresses

$\tilde{\alpha}_N$ = estimate of Weibull scale parameters from a previous test run of parts operated at manufacturer's rated conditions

$\tilde{\beta}_N$ = estimate of Weibull shape parameter from a previous test run of switches operated at manufacturer's rated conditions

$\tilde{\alpha}_A$ = estimate of Weibull scale parameter from a previous test run of switches operated at accelerated stress conditions

$\tilde{\beta}_A$ = estimate of Weibull shape parameter from a previous test run of switches operated at accelerated stress conditions

The algorithm is based on a transformation function which assumes a constant ratio of the hazard rates. Its derivation appears in Reference 298.

The following table summarizes the results of the use of the algorithm in estimating Weibul parameters at rated stress levels:

Inputs to Algorithms:		Source
$\tilde{\alpha}_N = 8.15 \times 10^9$	$\tilde{\beta}_N = 1.68$	RADC TR 65-46 (3 amps, 1 cycle/sec., 25°C)
$\tilde{\alpha}_A = 1.409 \times 10^6$	$\tilde{\beta}_A = 1.11$	RADC TR 65-46 (6 amps, 30 cycles/sec., 25°C)
$\tilde{\alpha}_A^* = 3.16 \times 10^6$	$\tilde{\beta}_A^* = 1.13$	RADC TR 66-425 (6 amps, 30 cycles/sec., 25°C)

New Normal Stress Weibull Parameter Estimates:

Observed	Calculated
RADC TR 66-425 (3 amps, 1 cycle/sec., 25°C)	Model No. 5
$\tilde{\alpha}^*_{0.975} = 6.5 \times 10^{33}$	
$\tilde{\alpha}^*_N = 1.73 \times 10^{17}$	1.817×10^{10}
$\tilde{\alpha}^*_{0.025} = 8 \times 10^{9}$	
$\tilde{\beta}^*_{0.975} = 5.40$	
$\tilde{\beta}^* = 2.73$	1.70
$\tilde{\beta}^*_{0.025} = 1.62$	

The values for $\tilde{\alpha}^*_N$ and $\tilde{\beta}^*_N$ calculated from the algorithm compare favorably with 95% confidence limits, and therefore it is assumed that the algorithm fits the relationships between failure characteristics at rated and accelerated stress levels.

All future accelerated tests on this exact same part type would be conducted at 6 amps contact current, 30 cycles/second and 25°C ambient temperature. The new $\tilde{\alpha}^*_A$ and $\tilde{\beta}^*_A$ would be entered into the algorithm with the values of $\tilde{\beta}_N$, $\tilde{\alpha}_N$, $\tilde{\beta}_A$, and $\tilde{\alpha}_A$. The solution of the algorithm for $\tilde{\beta}^*_N$ and $\tilde{\alpha}^*_N$ would yield estimates of parameters of a Weibull cumulative failure distribution from which one could estimate any reliability characteristic of interest regarding life at rated stress levels for the current lot of relays.

RADC 66-425 lists additional desirable stress levels which are suitable for satisfactory use for this relay type.

INSTRUCTIONS FOR USE

1. Test a random sample of relays to first miss failure and record the failure times. This step must be carried out at least once at manufacturers' rated load for each relay type and for each different manufacturer whose parts are to be evaluated. These test conditions should be 3 amps contact load, one cycle/second actuation rate, and 25°C ambient temperature for the type of relay described in this test method. This step may be omitted if the parts under test are the same as those studied in RADC TR 66-425 (see Section 3.0).

2. Plot the failure times on Weibull probability paper and estimate the rated stress shape and scale parameters ($\tilde{\beta}_N$ and $\tilde{\alpha}_N$).

3. Test a random sample of relays at accelerated stress levels (6 amperes contact load, 30 cycles/second actuation rate, and 25°C ambient temperature). Record the number of cycles to first miss failure. This step may be omitted if the relays are the same as those tested in RADC TR 66-425 (see Section 3.0).

4. Plot these failure times on Weibull probability paper to estimate the accelerated Weibull shape and scale parameters ($\tilde{\beta}_A$ and $\tilde{\alpha}_A$).

   Note: Steps 1 through 4 are necessary requisites for gathering historical data to be used in the algorithm with current accelerated test results.

5. Same as Step 3 except that relays are from current production run or lot which is to be evaluated. The parts must be from the same manufacturer and be built from the same specification as those tested in Steps 1 and 3.

6. Plot these failure times on Weibull probability paper to estimate $\tilde{\beta}_A^*$ and $\tilde{\alpha}_N^*$.

7. Use the algorithm to estimate $\tilde{\beta}_N^*$ and $\tilde{\alpha}_N^*$. From these construct a Weibull cumulative failure distribution which will yield estimates of any desired reliability characteristic of the current parts at rated stress levels.

LIMITATIONS/RANGE OF APPLICABILITY

The major limitation of this method is that it requires a test run at rated stress levels. If this can be accomplished in a reasonable time and specifications do not change rapidly, then the method can be applied. The method requires estimates of $\tilde{\alpha}_N$, $\tilde{\beta}_N$, $\tilde{\alpha}_A$, $\tilde{\beta}_A$ for relays by different manufacturers and also for relays with different ratings, or contact configurations.

REFERENCES: 297 and 298.

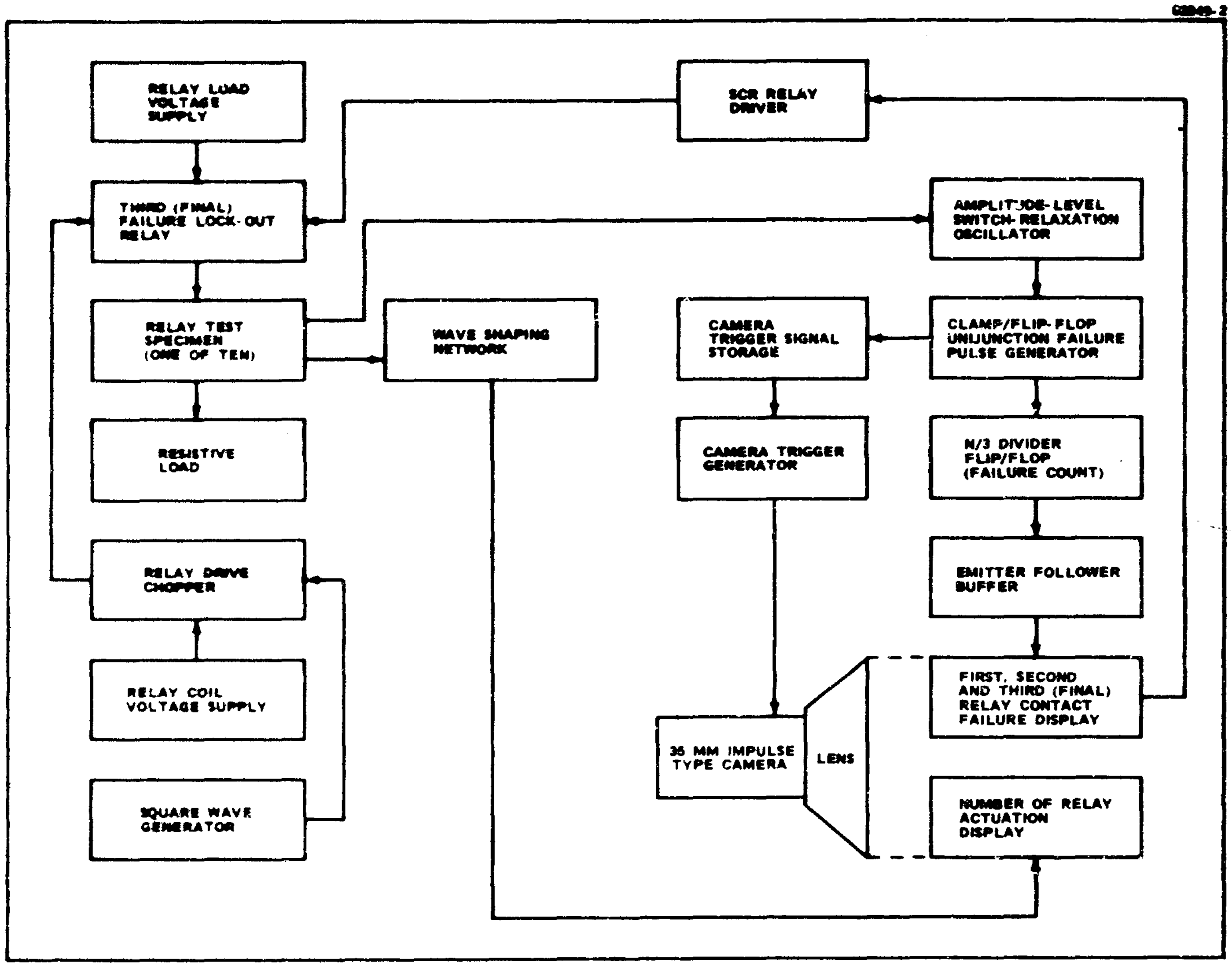

Figure 3.10.1. Schematic of Physical Test Method for Crystal Can Relays

3.11 PART NAME AND DESCRIPTION: Relay, Integral Capsular.

ALT Evaluation	
Assumptions Generally Accepted	X
Empirical Proof	
Algorithm	X
Physical Model	

This ALT method was used on a General Electric Integral Capsular Relay. It was a single pole, double throw configuration. The results obtained were based on tests performed on engineering test models.

SOURCE: This method was used by W. H. Lesser of General Electric in the final technical report for the U.S. Army Electronics Research and Development Agency on Contract DA 36-039 SC-78937, June 1963. (Reference 166.)

PURPOSE OF TEST: To estimate the parameters of the failure distribution of relays assuming that the failure times are distributed according to the Weibull distribution.

DEGREE OF VALIDATION

This series of tests has been performed only one time on this relay type and the tests were not completed at the time the report was written. Therefore it has not been proven that a repetition of this test program would yield similar significant effects and similar regression equations. The statistical basis on which the test method has been developed is sound and repeated use of the method will provide sufficient evidence for validation. Additional use will also provide information regarding the variation expected between manufacturing lots.

DESCRIPTION OF TEST METHOD

Special test panels were used in the performance of these tests. Nine relays were put on test at each combination of stresses. The accelerating stresses utilized were contact current, contact voltage, ambient temperature, coil voltage, and actuation rate. The test equipment was designed to stop every time a failure occurred in any contact of any of the relays on test and lights were used to indicate the location of the failure. Failure was defined as the following: failure to open, failure to close, failure of coil to conduct current, excessive contact resistance. The analysis was based on cycles to first miss failure. A more detailed description of the test method appears in Reference 453.

SUMMARY OF RESULTS

A central composite experimental design was used in evaluating the accelerating stresses and stress levels which affected the number of cycles to first miss failure. Twenty-seven test runs of nine relays each were tested to first miss failure (although the report was written before all relays had failed in certain test runs). The underlying distribution of

failure times was Weibull in most cases. The Weibull parameters were estimated for each set of combined stresses and these estimates were used to calculate a regression equation characterizing Weibull parameter change as related to the severity of accelerating stress.

The following regression equations are those developed for use in estimating β (the Weibull shape parameter) and ln y (the natural logarithm of the Weibull characteristic life).

$$\begin{aligned}\beta = {} & .9596 + .3075X_1 - .2991X_1X_3 \\ & - .3071X_1X_4 - .2941X_1X_5 - .3315X_2X_3 \\ & - .3075X_2X_5 + .4283X_3X_4 + .2763X_3X_5\end{aligned} \tag{3.11.1}$$

$$\ln y = 7.5095 - 1.8154X_1 \tag{3.11.2}$$

where

$$X_1 = \frac{\ln\text{ (Contact Current in amps)}}{.464} \tag{3.11.3}$$

$$X_2 = \frac{\text{Contact voltage (volts)} - 28}{7} \tag{3.11.4}$$

$$X_3 = \frac{\text{Ambient temperature (°C)} - 75}{50} \tag{3.11.5}$$

$$X_4 = \frac{\text{Coil Voltage (volts)} - 28}{7} \tag{3.11.6}$$

$$X_5 = \frac{\ln\text{ (Rate of Operation in cycles/minute)} - 4.094}{.695} \tag{3.11.7}$$

These formulae can be used to estimate the life characteristics of this relay type by inserting the correct operating or environmental levels into them and solving for the values of X_1 through X_5. The X values can then be inserted into formula (3.11.1) to solve for β at that given combination of stresses and into formula (3.11.2) to calculate ln y. In order to construct a cumulative failure distribution on Weibull probability paper $\ln \alpha$ is required where α is the Weibull scale parameter. This can be calculated by the formula

$$\ln \alpha = \beta \ln y \tag{3.11.8}$$

Then with the estimate of β and $\ln \alpha$ the cumulative failure distribution can be prepared and any reliability can be calculated for a given combination of either rated or accelerated stress levels.

The manner in which these mathematical models (the regression equations) can then be used as an ALT method is as follows. For this relay type one can use the given equations to calculate β and $\ln \alpha$ and then draw the cumulative failure distribution on Weibull probability paper. This same procedure could be followed for any selected combination of accelerated stresses (within the ranges of those used in the development of this method). Then a current lot of these parts could be life tested at the defined levels of accelerated stresses and their failure times could be plotted on Weibull probability paper. This plot could be used in estimating an α and β. If these Weibull parameter estimates agreed reasonably well with those same values generated from the regression equations, the parts being evaluated would qualify to have their life characteristics at normal conditions calculated by means of the regression equations.

INSTRUCTIONS FOR USE

This is a step-by-step instruction for using this ALT method:

1. Calculate β and $\ln y$ for manufacturer's rated conditions from the regression equations. With these results calculate $\ln \alpha$ from the relationship $\ln \alpha = \beta \ln y$.

2. Calculate β and $\ln y$ for some combination of accelerated stresses using the regression equations. With these values calculate $\ln \alpha$ for the accelerated conditions by the relationship $\ln \alpha = \beta \ln y$.

3. Select a sample of relays of this type at random and test them to first miss failure at the combined accelerated stress levels used in Step 2 above and record the failure times in cycles (see Section 3.0).

4. Plot the failure times on Weibull probability paper and calculate β and $\ln \alpha$.

5. If the estimates of β and $\ln \alpha$ from Step 2 (theoretical accelerated estimates) and Step 4 (observed accelerated estimates) are "reasonably" in agreement then the results of Step 1 (theoretical rated estimates) can be used for estimating the life characteristics of the relays on hand as if they had been operated at rated stress levels. With β and $\ln \alpha$ from Step 1 a cumulative failure distribution can be constructed on Weibull probability paper and it will yield any desired reliability estimate.

LIMITATIONS/RANGE OF APPLICABILITY

The major limitation of this method is that when the observed results at accelerated stresses are compared with the theoretical results based on the same stress levels there is no quantitative measure for determining when the two sets of results agree. This limitation could be overcome

by the calculation of confidence limits for each of the parameters estimated by the regression equations. With regard to range of applicability this general method has been used on other relay types but the actual regression equations derived as well as the definitions of failure are different. Therefore, these algorithms for calculating Weibull parameters at various stress levels have been developed only for use on the General Electric Integral Capsular Relays.

REFERENCES

166 and 453.

3.12 PART NAME AND DESCRIPTION: Relay, Power

ALT Evaluation	
Assumptions Generally Accepted	X
Empirical Proof	X
Algorithm	X
Physical Model	

This ALT method was used on Struthers-Dunn Relay Type FC-215. It is a double pole, double throw switching relay in a miniature configuration. It is hermetically sealed and has a contact rating of 10 amperes resistive load at 125°C and 30 volts DC.

SOURCE: This ALT method was developed for this application by W. J. Fontana of the U.S. Army Electronics Command. (Reference 128.) The data he used in demonstrating the use of the method was generated by J. Jord and G. Mease of Struthers-Dunn. (Reference 238.)

PURPOSE OF TEST: To estimate the parameters of the failure distribution of relays assuming that the failure times are distributed according to the Weibull distribution.

DEGREE OF VALIDATION

Two test programs have been performed using this test method on Struthers-Dunn FC-215 relays. In both cases 10 relays were tested at each of 15 combinations of accelerated stresses. The exact same stress levels were applied and the results obtained from the two programs were in close agreement. The results of these two programs are reported in detail in References (128) and (238). Therefore it appears that this method has been validated at least within the ranges of the stress levels used. These ranges were 7 to 13 amps contact load, 5 to 60 cycles/minute actuation rate and 25 to 125°C ambient temperature. The curves shown in Figures 3.12.1 and 3.12.2 are not in complete agreement with the regression equations but represent lines of best fit drawn through the observed values. The differences are probably due to sampling error. Therefore this test method and associated analysis method can be expected to give indications of whether or not to use the regression equations for calculating expected life at rated stress levels. If, however, the results of an accelerated test differ from the expected results (from the regression equations) it is not possible to determine an estimate of rated life.

DESCRIPTION OF TEST METHOD

All parts tested came from a single manufacturing lot. They were randomly subdivided into 15 groups of 10 relays each for testing at various levels of combined stresses. Leads were soldered to the relays and each group of 10 parts was put on test in recorder monitored, temperature controlled connection ovens. Each fixed contact was monitored continuously and the duty cycle was 50% on, 50% off. The definition of failure

was either a catastrophic failure or a contact miss which was defined as a contact closure in which the voltage drop exceeded 2 volts and/or less than 28 volts dc on opening. The tests were continued until all 10 relays in the test run had reached end of life. End of life was defined as catastrophic failure to operate, or three consecutive misses or a miss rate greater than 5 per 1000 cycles in any given period. The accelerating stresses applied were contact current, actuation rate, and ambient temperature. The stresses were applied in combination. The stress levels used were both greater than and less than manufacturer's rated levels.

SUMMARY OF RESULTS

The study used in the development of this ALT method took the form of a central composite experimental design. The details of its use are given in Reference (128). Fifteen test runs of 10 parts each were tested to total failure. The failure times were found to fit the Weibull distribution. The Weibull parameters were estimated for each test run and these estimates were used to calculate a regression equation characterizing parameter change as related to magnitude of accelerating stress.

The following are the regression equations derived to calculate various Weibull parameters when these relays are tested using contact current, actuation rate and ambient temperature as accelerating stresses:

The following are the regression equations derived to calculate various Weibull parameters when these relays are tested using contact current, actuation rate and ambient temperature as accelerating stresses:

$$\ln\alpha = 4.4. - 1.37X_1 - .03X_2 - .25X_3 - .42X_1^2 - .63X_2^2 - .98X_3^2 + .56X_1X_2 - .30X_1X_3 - .10X_2X_3 \quad (3.12.1)$$

$$\beta = 3.62 - .37X_2 - .62X_1^2 \cdot .74X_3^2 \quad (3.12.2)$$

$$\mu = 3.30 - 2.26X_1 + .71X_2 + .46X_3 + 82X_1^2 - .60X_1X_3 \quad (3.12.3)$$

$$\rho(.95) = 1.58 - .60X_1 - .45X_3^2 \quad (3.12.4)$$

$$\rho(.90) = 1.92 - .83X_1 - .45X_3^2 \quad (3.12.5)$$

$$\rho(.80) = 2.31 - 1.16X_1 - .44X_3^2 \quad (3.12.6)$$

where

$\ln \alpha$ = the natural log of the Weibull scale parameter

β = the Weibull shape parameter

μ = mean life of the Weibull distribution

ρ = reliable life at probability .95, .90, and .80

$$X_1 = \frac{\text{Contact current (amperes)} - 10}{3} \qquad (3.12.7)$$

$$X_2 = \frac{\text{Actuation rate (c.p.m.)} - 32.5}{18.5} \qquad (3.12.8)$$

$$X_3 = \frac{\text{Ambient temperature (°C)} - 75}{50} \qquad (3.12.9)$$

These formulae can be used to estimate the life characteristics of this type of relay by inserting the contact current, actuation rate and ambient temperature into Equations 3.12.7, 3.12.8 and 3.12.9 respectively and solving for X_1, X_2, and X_3. These three values then can be inserted into Equation 3.12.2 to solve for β (the Weibull shape parameter). As an example, suppose it is desired to operate this part at 13 amps contact current, 20 cycles/minute actuation rate, and at 25°C ambient.

$$X_1 = \frac{13-10}{3} = 1$$

$$X_2 = \frac{20-32.5}{18.5} = -.68$$

$$X_3 = \frac{25-75}{50} = -1$$

Then

$$\beta = 3.62 - .37\ (-.68) - .62\ (1)^2 - .74\ (-1)^2 = 2.51$$

The other formulae could be utilized and they would yield a full description of the underlying failure distribution of failure times for parts operated at these stress levels.

Figures 3.12.1 and 3.12.2 are examples of approximate curves which can be used in place of the regression equations for rough estimates.

It must be noted that with the regression equations (or the curves) the manner in which this ALT would be used would be to test a group of parts at some overstress combination of contact current, actuation rate,

and ambient temperature. Then the failure times would be plotted on Weibull probability paper in order to estimate $\ln \alpha$ and β. These estimates of the Weibull parameters would be compared to their counterparts calculated from the regression equations or taken from the curves. If the two sets of results were in substantial agreement, then this would qualify the use of the regression equations for calculating estimates of Weibull parameters at rated stress levels. The definition of "substantial agreement" would have to be agreed on prior to the performance of the accelerated test program.

INSTRUCTIONS FOR USE

The following is a step-by-step instruction for using this test method:

1. Calculate $\ln \alpha$ and β for manufacturers rated conditions from the regression equations.

2. Calculate $\ln \alpha$ and β from the regression equations for some combination of accelerated stress levels using the regression equations.

3. Select relays at random, test them to failure at the same accelerated stress levels assumed in Step 2 and record the failure times in cycles (see Section 3.0).

4. Plot the failure times on Weibull probability paper and calculate β and $\ln \alpha$.

5. If the estimates of β and $\ln \alpha$ from Step 4 are "reasonably close" to each other, then the values of $\ln \alpha$ and β from Step 1 qualify for use in the estimation of life at rated stress levels. This can be accomplished by either using the regression equations for μ, $\rho(.95)$, etc. or by preparing a chart on Weibull probability paper to depict the cumulative failure distribution at rated conditions.

LIMITATIONS/RANGE OF APPLICABILITY

The major limitation of this AIT method is that there are no definite rules for determining when test results agree with the theoretical results as calculated from the regression equations. This limitation could be overcome by the calculation of confidence limits for each of the parameters estimated by the regression equations. With regard to range of applicability this general method has been used on other relay types but the actual regression equations derived are different. Therefore, these algorithms for calculating Weibull parameters at various stress levels have been proven only for use of Struthers-Dunn FC-215 relays.

REFERENCES

128 and 238.

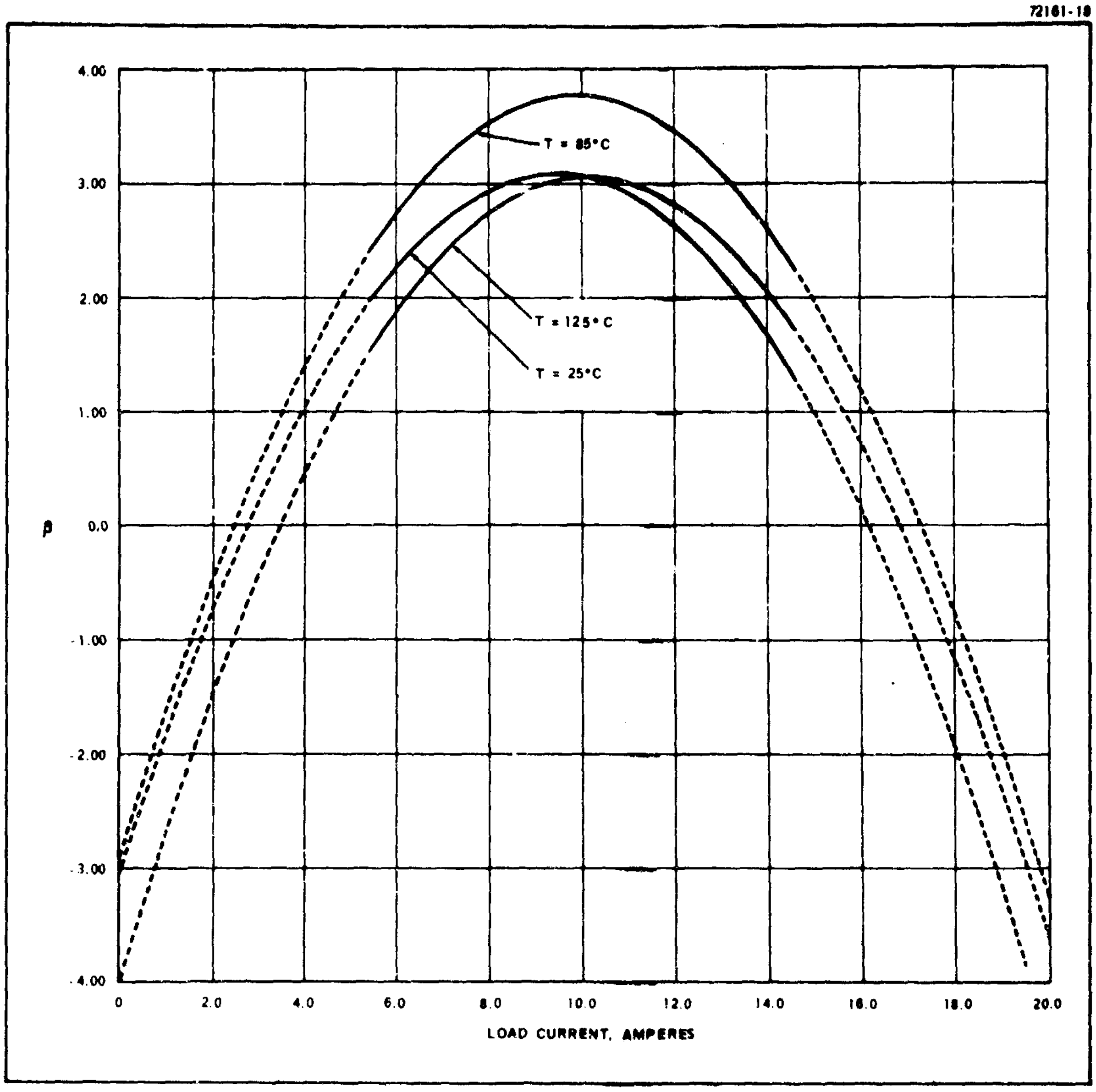

Figure 3.12.1. Weibull Slope Parameter (β) at N = 20 CPM

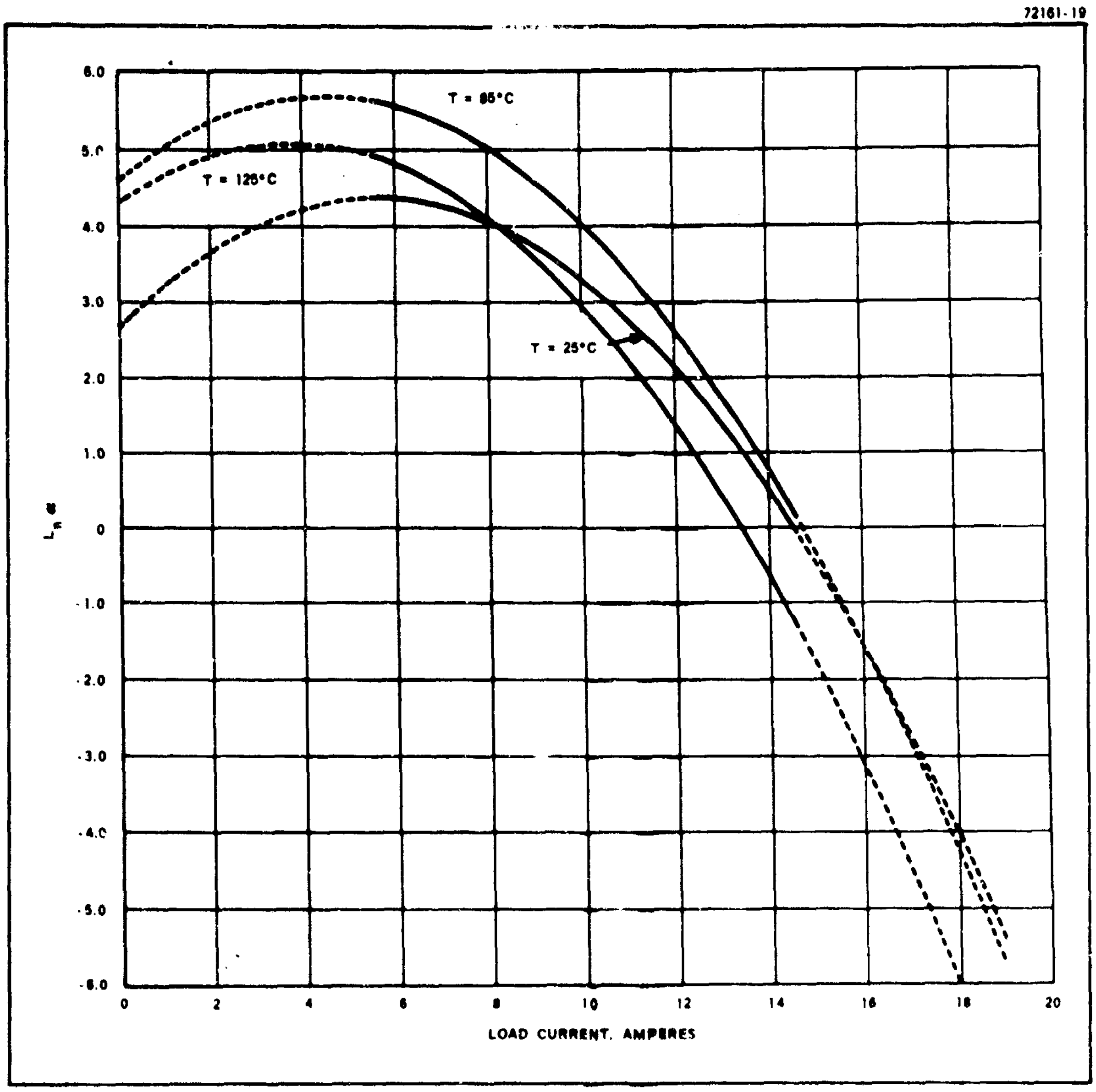

Figure 3.12.2. Ln of Weibull Scale Parameter (Ln α) at N = 20 CPM (Scale Factors = 100,000 Cycles)

3.13 PART NAME AND DESCRIPTION: Switch, Limit

Contact rating, manufacturer, contact configuration unspecified.

ALT Evaluation	
Assumptions Generally Accepted	___
Empirical Proof	___
Algorithm	X
Physical Model	___

SOURCE: The information presented here to describe this ALT method was published in Reference 15 "Reliability Demonstration Using Overstress Testing" by D. A. Authment, C. E. Foshee, F. C. Herzner, and C. M. Orr of ARINC in July 1965.

PURPOSE OF TEST: To reduce reliability demonstration test time by estimating MTBF in a manner that will always result in a conservative estimate.

DEGREE OF VALIDATION

This method is not meant to result in an exact estimate of the reliability parameter of interest but only one that has an unknown safety factor in it. It has been tried on several sets of data from several different sources which were generated on several different parts and materials. It does reduce test time, but unfortunately in doing so it gives no inkling of the size of the resulting safety factor. The general theory and assumptions on which the method are based seem reasonable enough; it does reduce test time, but in doing so it deprives the user of all quantitative information regarding the reliability measures he is trying to demonstrate. Hence, it is doubtful if the reduction in test time is economical since little information is furnished the user.

DESCRIPTION OF TEST METHOD

The details of the test method used and the definition of failure were not available since the method was used on data that were generated in another study (Reference 69).

SUMMARY OF RESULTS

The method is based on the assumption that a stress versus life (S-N) curve can be developed for the part of interest. It further assumes that the S-N curve is of such a nature that as the stress level is decreased, mean life of the parts increase. It recognizes that probably not enough parts are available for demonstration testing at each selected stress level to result in a valid statistical estimate of the distribution of failure times.

Briefly, it is suggested that two overstress levels be selected for testing a small sample of parts to failure. The MTBF of the sample at each stress level is calculated. The two calculated points are plotted on a chart that has mean cycles to failure as the abscissa and stress

as the ordinate. A straight line is drawn through the two points. Where it intersects the stress level designated as mission stress level or manufacturers' rated stress is assumed to be a conservative estimate of the MTBF at that stress. This theory is shown diagrammatically in Figure 3.13.1. The conservativeness of the MTBF estimate is assured due to the assumed curvature of the S-N curve.

Note: The authors state that usually the abscissa is plotted in logarithm of failure time. However, the logarithmic transformation is usually used to convert the S-N curve to a straight line which defeats the purpose of the method. Therefore, it is believed that the abscissa should be plotted in actual cycles to failure.

INSTRUCTIONS FOR USE

The following instructions for using this ALT method are illustrated with the aid of data used on limit switches in Reference 15.

1. Divide the switches available for the reliability demonstration test into two groups by some randomizing process.

2. Select two stress levels which are above the stress expected during normal usage of the switches. The stresses and stress levels should be based on past test data and experience so that they will result in a substantial reduction in test time without radically changing the expected failure mode. In the case of limit switches the parts selected were rated at 10 amperes contact current. They were tested at 30 and 60 amperes.

3. The mean cycles to failure for both stress levels are plotted on the abscissa of an S-N curve with contact current in amperes on the ordinate. A straight line is drawn through the data points at 60 and 30 amperes. Where it intersects the 10 ampere ordinate a line is dropped vertically to the mean life axis as in Figure 3.13.2.

4. This becomes the estimate of the mean life of the switches if they had been operated at 10 amperes.

5. If sufficient historical data is available on this exact same part type operated at 10 amperes the mean of the historical data can be compared with the mean obtained in Step 4. Estimates of the error in estimation of mean life can then be made and the test time saved can be recorded.

LIMITATIONS/RANGE OF APPLICABILITY

While the assumptions on which the method is based seem plausible, it does not appear that the time saved yields the user of the demonstration test very much information unless he has a wealth of data at rated stress levels. Even then the comparison is very qualitative. The

assumptions regarding the S-N curve's abscissa as logarithm of time does not fit the majority of life test data. Usually the logarithmic transformation is used to transform the S-N curve into a straight line. In fact, in the numerical example on limit switches the historical data at 10 amperes plotted as a straight line and it is not apparent how the estimate of MTBF based on a line through the overstress data points was computed. Another weakness of the data used in the limit switch example was that the mean cycles to failure were computed only on the parts which failed.

The suggested ALT method was tried on 10 different parts and materials and these results appear also in Reference 15. They ranged from limit switches to airplane wings and tails as well as many material coupons and specimens.

REFERENCE: 15

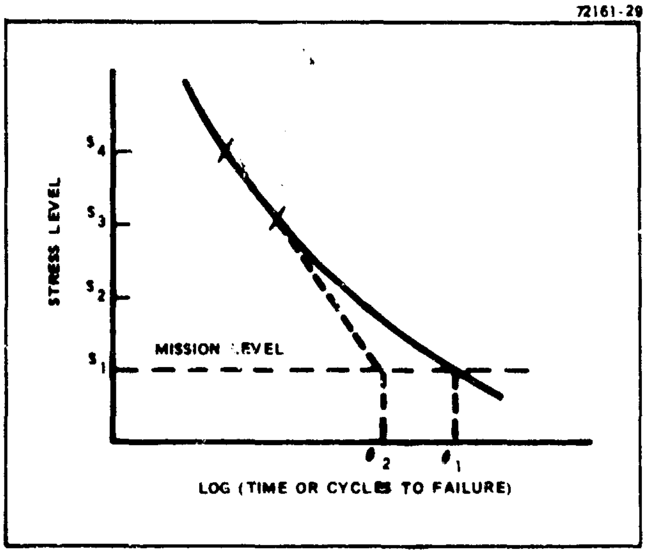

Figure 3.13.1. Typical S-N Curve

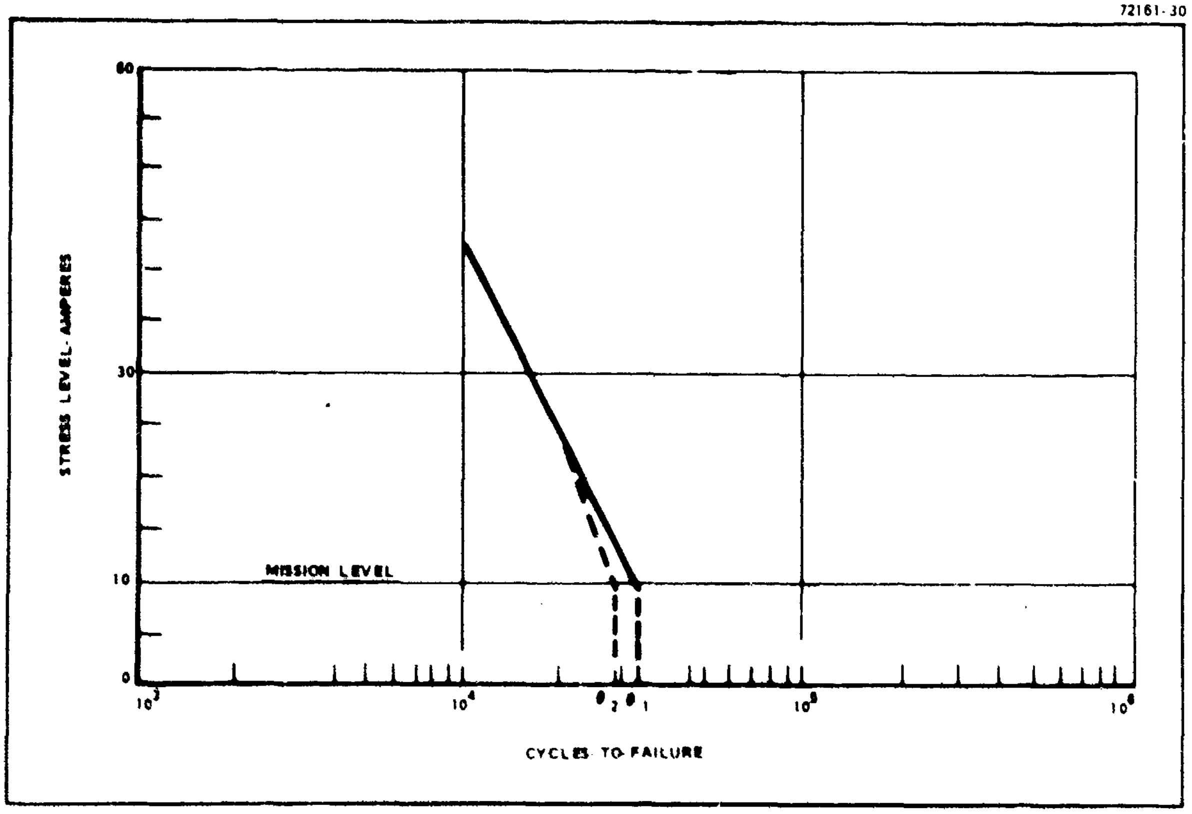

Figure 3.13.2. Test-to-Failure Electromechanical Limit Switch

3.14 PART NAME AND DESCRIPTION: Switch, Snap Action, Subminiature

ALT Evaluation	
Assumptions Generally Accepted	X
Empirical Proof	X
Algorithm	X
Physical Model	

The parts studied were designated as switch type M8805/2-1. They were produced by four different manufacturers, but the identity of the manufacturers is not given in the referenced reports. The switches have a contact rating of 5 amperes, are designed for a temperature range of from -65°F to 250°F, and have contacts of fine silver. The operating force at the plunger is specified as 5.3 ounces maximum and the release force is 1.4 ounces minimum. The nominal contact overtravel is .007 inches.

SOURCE: The subject ALT method was developed for RADC by Hughes Aircraft Company, Fullerton, California in the study programs reported in References 297 and 298.

PURPOSE OF TEST: To develop an ALT method based on time transformations of the distribution functions of parts tested to first miss failure which will yield quantitative estimates of reliability characteristics at rated stresses.

DEGREE OF VALIDATION

There is good evidence to verify that the ALT method developed here is valid since two sets of test data yield comparable results. The tests were performed about a year apart, on switches produced by the same manufacturer, using the same test method and the exact same stress levels. The weakness is that a larger sample size would have resulted in a more concrete validation of the results. No physical model has yet been found to explain the results.

DESCRIPTION OF TEST METHOD

Ten switches were tested simultaneously on a specially designed test equipment using three four lobe cams to actuate the switch plunger. The cams were designed so that the switches had a duty cycle of 70% on, 30% off. The switches were loaded by means of fixed non-inductive wire wound resistors and variable wire wound resistors. The resistor load bank was installed in an evaporative cooling bath of Freon 113. Synchronous motors and a combination of gears were utilized to establish the desired actuation rate. Failure times were recorded automatically by photographing a display board with a 16mm movie camera. The camera was actuated by automatic miss detection equipment and one frame was taken for each miss on each switch until ten failures were recorded on each switch being tested.

In addition to recording the number of cycles to tenth miss failure, contact bounce, contact resistance, operate force, release force, and contact overtravel were measured prior to test and periodically as the test progressed. Figure 3.14.1 is a schematic diagram of the test equipment used.

SUMMARY OF RESULTS

The statistical experiment consisted of a full factorial experiment with 10 replications which investigated the effects on switch life of contact current, actuation rate, contact overtravel, and manufacturer as well as their interactions when applied in combination. One cell of the experiment was defined as manufacturer's rated stress levels while all others were combinations containing some or all overstress conditions.

An analysis of variance was conducted with failure defined as first miss as well as the definition at 10th miss and substantially similar results were found. The first order interactions between actuation rate and contact overtravel, contact load and manufacturer, and contact overtravel and manufacturer were all statistically significant at the $F_{.01}$ level.

The underlying distribution of failure times at both rated and accelerated stresses was Weibull.

Ten parts were tested at rated stresses and the results reported in each of two successive studies (Reference 297 and 298). Rated stress was defined as a contact current of 5 amperes, actuation rate of 70 cycles/minute and a contact overtravel of .005 inches. In both studies accelerated test runs were made at identical stresses of 10 amps contact current, 300 cycles/minute actuation rate, and .010 inches contact overtravel. The results of the two sets of results at identical rated and accelerated stresses were inserted into the following algorithms:

$$\tilde{\alpha}_N^* = \left(\frac{\tilde{\alpha}_N \, \tilde{\beta}_A}{\tilde{\alpha}_A \, \tilde{\beta}_N}\right) \frac{\tilde{\alpha}_A^* \, \tilde{\beta}_N^*}{\tilde{\beta}_A^*} \qquad \tilde{\beta}_N^* = \tilde{\beta}_A^* + (\tilde{\beta}_N - \tilde{\beta}_A)$$

where:

$\tilde{\alpha}_N^*$ = estimate of Weibull scale parameter for parts if they had been tested at manufacturer's rated operating and environmental stresses

$\tilde{\beta}_N^*$ = estimate of Weibull shape parameter for parts if they had been tested at manufacturer's rated operating and environmental stresses

$\tilde{\alpha}_A^*$ = estimate of Weibull scale parameter obtained from a current test run at accelerated stresses

$\tilde{\beta}_A^*$ = estimate of Weibull shape parameter obtained from a current test run at accelerated stresses

$\tilde{\alpha}_N$ = estimate of Weibull scale parameters from a previous test run of parts operated at manufacturer's rated conditions

$\tilde{\beta}_N$ = estimate of Weibull shape parameter from a previous test run of switches operated at manufacturer's rated conditions

$\tilde{\alpha}_A$ = estimate of Weibull scale parameter from a previous test run of switches operated at accelerated stress conditions

$\tilde{\beta}_A$ = estimate of Weibull shape parameter from a previous test run of switches operated at accelerated stress conditions.

The algorithms are based on a transformation function which assumes a constant ratio of the hazard rates. Its derivation appears in Reference 298.

The following table summarizes the results of the use of the algorithms in estimating Weibull shape and scale parameters at rated stress levels:

Inputs To Algorithms:		Source
$\tilde{\alpha}_N = 5.38 \times 10^{21}$	$\tilde{\beta}_N = 4.07$	RADC TR 65-46 (5 amps, 70~/minute, 0.005 inches)
$\tilde{\alpha}_A = 1.44 \times 10^4$	$\tilde{\beta}_A = 1.00$	RADC TR 65-46 (10 amps, 300~/minute, 0.010 inches)
$\tilde{\alpha}_A^* = 4.04 \times 10^5$	$\tilde{\beta}_A^* = 1.293$	RADC TR 66-425 (10 amps, 300~/minute, 0.010 inches)

New Normal Stress Weibull Parameter Estimates:

	Observed	Calculated
	RADC TR 66-425 (5 amps, 70~/minute, 0.005 inches	Model No. 5
	$\alpha_{0.90}^* = 8 \times 10^{26}$	
	$\tilde{\alpha}_N^* = 2.82 \times 10^{16}$	1.25×10^{23}

RADC TR 66-425
(5 amps, 70~/minute,
0.005 inches)

Model No. 5

$\alpha^*_{0.10} = 5 \times 10^2$

$\beta^*_{0.90} = 5.31$

$\tilde{\beta}^*_N = 3.228$ 4.36

$\beta^*_{0.10} = 2.28$

INSTRUCTIONS FOR USE

1. Test a random sample of switches to first miss failure at rated combined stresses consisting of 5 amps contact current, 70 cycles/minute actuation rate, and a contact overtravel of .005 inches (see Section 3.0). Plot the failure times on Weibull probability paper and estimate the shape and scale parameters by the usual methods. This results in the development of $\tilde{\alpha}_N$ and β_N for use in the algorithm.

2. Test a random sample of switches to first miss failure at accelerated combined stresses consisting of 10 amps contact current, 300 cycles/minute actuation rate, and a contact overtravel of .010 inches (see Section 3.0). Note that RADC TR 66-425 gives other desirable combinations of accelerated stress levels. Plot the failure times on Weibull probability paper and estimate the shape and scale parameters by the usual methods. This results in the development of $\tilde{\alpha}_A$ and β_A for use in the algorithm.

3. In the future to estimate any reliability characteristic at rated stress levels, it is only necessary to run the accelerated test as in Step 2, estimate $\tilde{\alpha}^*_A$ and $\tilde{\beta}^*_A$, combine these estimates of the Weibull scale and shape parameters with those generated in Steps 1 and 2 above in the algorithms in order to gain estimates of $\tilde{\alpha}^*_N$ and β^*_N the estimates of Weibull parameter estimates at rated stresses.

LIMITATIONS/RANGE OF APPLICABILITY

One drawback of this method is that it is only useful on a part whose life at rated conditions is short enough to allow the performance of a test at rated stress levels, since this furnishes required inputs to the transformation algorithm. The method and the algorithm are useful on switches of this type produced by at least four manufacturers; however, the inputs to the algorithm are not transferable between different manufacturer's parts. This means that tests at rated stresses

must be performed for each different manufacturer's parts. There is no evidence that the ALT method is useful on other switch types and configurations since no other types have been tested.

REFERENCES: 297, 298.

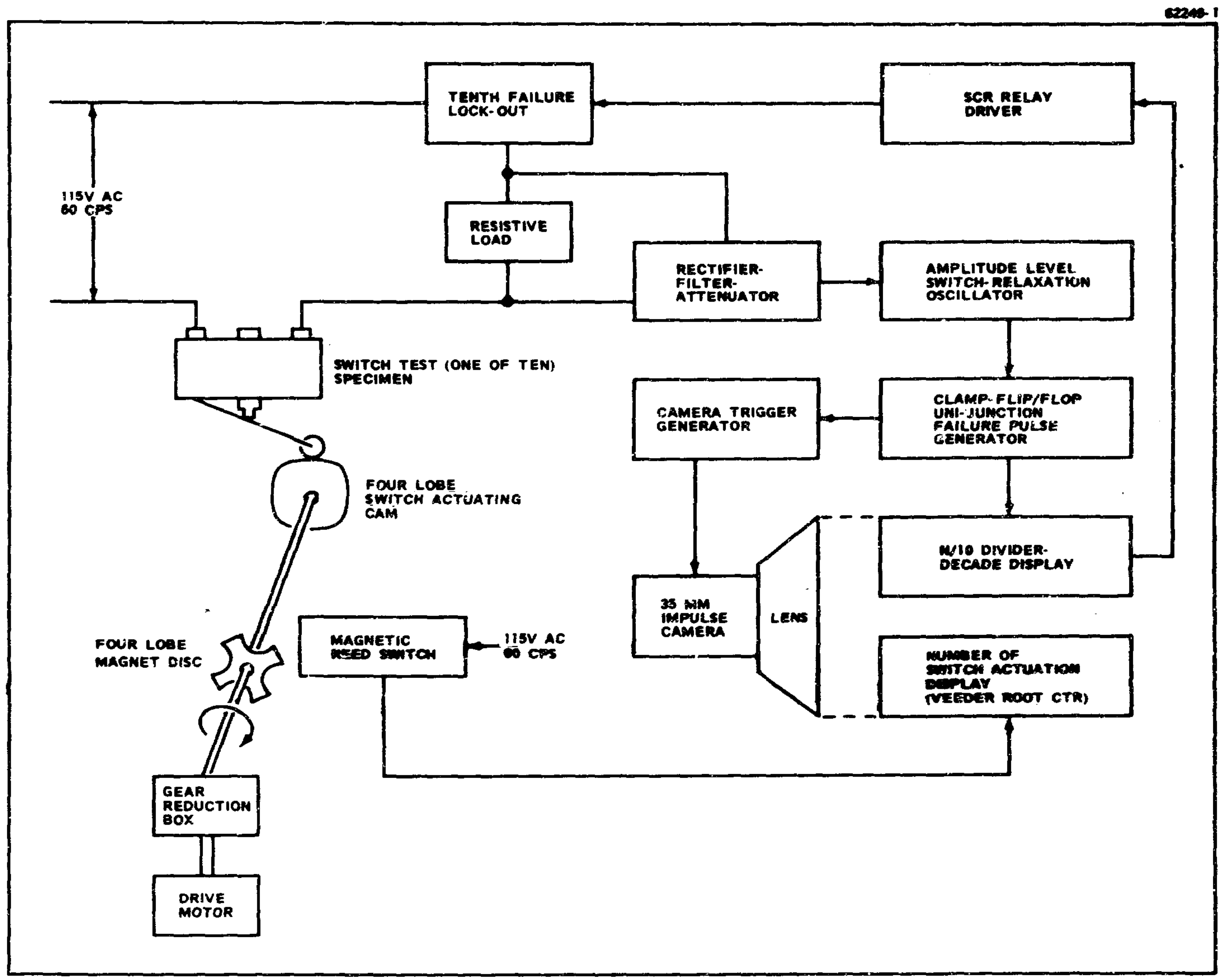

Figure 3.14.1. Schematic of Physical Test Method for Snap Action Switches

3.15 PART NAME AND DESCRIPTION: Resistors, Carbon Composition.

ALT Evaluation	
Assumptions Generally Accepted	
Empirical Proof	
Algorithm	X
Physical Model	X

Tests were performed on three types of resistors as follows: 10K ohms, 1/2 watt; 270K ohms, 1/2 watt; 1000 ohm, 1/2 watt

SOURCE: This test method was reported by R. E. Thomas and H. C. Gorton of Battelle Memorial Institute in Physics of Failure in Electronics, Volume 2.

PURPOSE OF TEST: Estimation of Mean Life in Hours

DEGREE OF VALIDATION

The report upon which the formulation of these accelerated life tests was based represents a single performance of a test program. There is no proof given in the literature that this same part type would yield the same results if the tests were duplicated. There is a statistical shortcoming in the preparation of the Arrhenius plot which the author mentions. That is the fact that only three slopes were available for calculating the slope B to be used in the mathematical expression for the acceleration factor. A considerably larger number of estimates of the relationship between the natural logarithm of the degradation rates and the reciprocal of the absolute temperature would be desirable in order to be able to state with some assurance that these data really fit the Arrhenius Model.

DESCRIPTION OF TEST METHOD

For the development of this accelerated test method, 120 resistors were tested at each of three ambient temperatures. Rate of degradation was measured as the resistance change with time. The three levels of thermal stress which were employed were 70, 100 and 125°C. The 70°C level was defined as normal stress. Measurements of the change in resistance were recorded after 90, 250, 506, 743, 999, 1500 and 2,000 hours of operation.

SUMMARY OF RESULTS

This accelerated test method is based on the assumption that the change in resistance over time which is due to temperature is in accordance with the Arrhenius Model. The Arrhenius Model uses the following formula for the calculation of equivalent life at rated temperature:

$$t = e^{-B\left(\frac{1}{T'} - \frac{1}{T}\right)} t' \qquad (3.15.1)$$

where t = life at rated temperature

t' = life at accelerated temperature

B = an empirical constant

T' = accelerated ambient temperature in °K

T = normal ambient temperature in °K

The following steps represent a numerical example describing the use of this model in the accelerated life testing of the 10K resistors:

1. The resistance of the 120 resistors in a test cell were recorded initially and their arithmetic mean was calculated. This same procedure was carried out after 90, 250, 506, 743, 999, 1500 and 2,000 hours of operation. Test cells were operated at 70, 100 and 125°C.

2. A regression line was calculated for each test cell which estimated the mean resistance change with time. The regression line was calculated by the method of least squares. For the three test cells the following linear equations were found to represent the mean change of resistance with time:

$$R_{70} = 9904 - .0713t$$
$$R_{100} = 9854 - .0736t$$
$$R_{125} = 9453 - .9992t$$

3. An Arrhenius Plot was prepared by charting the natural logarithms of the slopes from the above three equations with their respective reciprocals of the ambient temperature in °K. A regression line was fit to these three points by the method of least squares. The resulting line had the equation

$$\ln R = 16.24 - 6500 \frac{1}{T}$$

4. The acceleration factors were then calculated from the equation:

$$\tau = e^{-B\left(\frac{1}{T'} - \frac{1}{T}\right)} \qquad (3.15.2)$$

$$\tau = e^{-6500\left(\frac{1}{373} - \frac{1}{343}\right)}$$

$$\tau = 3.837$$

The numbers in the above example demonstrate the calculation of the acceleration factor for parts tested at the accelerated thermal stress of 100°C where 70°C is defined as the rated stress level.

5. The acceleration factors for the whole range of overstress thermal testing were generated by passing a smooth curve through the three acceleration factors calculated for operation at 70, 100 and 125°C. These are not presented because the authors indicated that "the numerical values of the acceleration factors are not to be taken as established values." (Reference 321.)

INSTRUCTIONS FOR USE

1. Place n resistors on test. At m sequential time intervals measure the n resistances and average them. This will yield m averages.

2. Determine the slope of the linear regression line through the averages of these observations by the method of least squares. This should be done for levels of thermal stress. The dependent variable is mean resistance, and the independent variable is time. Thus, k slopes will be calculated, each at one of k temperatures.

3. Make an Arrhenius Plot by preparing a graph with the natural logarithm of the rate of degradation (i.e., the slope of the regression line of Step 2 above) as the ordinate and the reciprocal of the absolute temperature as the abscissa. This is done for each thermal stress level.

4. Determine if true acceleration exists by drawing a linear regression line through the points on the Arrhenius Plot. If the points can be reasonably represented by a straight line, it can then be assumed that the Arrhenius Model represents the degradation of the resistance with time. The slope of this linear regression line is B.

5. The acceleration factors are then computed using formula 3.15.2. The operating hours at accelerated thermal stress levels can then be converted to equivalent hours at normal temperatures with formula 3.15.1.

LIMITATIONS/RANGE OF APPLICABILITY

The major limitation regarding the use of this method is the accuracy of the Arrhenius Plot. In order to determine if true acceleration exists and if the Arrhenius Model faithfully represents the degradation which occurs, it is necessary to have as many points as possible for calculating the B factor in the formula for acceleration rate. Regarding range of application, it appears from the data presented that resistors having different ratings require different acceleration factors. It is not apparent from the data presented whether the same rated part produced by different manufacturers could use the same acceleration factors. The manufacturer whose parts were used in the examples shown was not given. Therefore, the user of this method must go through all the steps required to calculate his own acceleration factors for a given part type. No statistical test is used in either of the linear

regressions to indicate the rules for goodness of fit.

The use of the Arrhenius Model is based on the requirement that generally the failure times at both accelerated and at rated stress levels must be from the same distribution family. A notable exception to this rule occurs with the Weibull distribution. In this case there is the added requirement that the shape parameters at both accelerated and rated stresses must be equal for the Arrhenius Model to hold.

REFERENCES

320 and 321.

3.16 PART NAME AND DESCRIPTION: Transistor Types 2N332-336

ALT Evaluation	
Assumptions Generally Accepted	___
Empirical Proof	___
Algorithm	X
Physical Model	X

The manufacturer of the parts is not given. They were selected from a regular production run and oven baked prior to testing.

SOURCE: This test method and its results were reported by R. E. Thomas and H. C. Gorton of Battelle Memorial Institute in several publications in 1964.

PURPOSE OF TEST: Estimation of Mean Life at Rated Stress Levels.

DEGREE OF VALIDATION

This method has not been validated because the authors of the method state - "The numerical values of the acceleration factors are not to be taken as established values. The primary purpose of these applications consists of emphasizing the method, not the primary results", Reference 321. Therefore, any user of this method will have to first run a series of tests to determine his own acceleration factors for this part type. The line of best fit for the Arrhenius plot was fit through only three points in the data presented in References 320 and 321. A more accurate estimate of the line could probably be made with a substantially greater number of points.

DESCRIPTION OF TEST METHOD

The parts tested were from a single lot and they were oven baked prior to classification and testing. When operated under power they were cycled at 50 minutes on and 10 minutes off. Rate of degradation was measured by logarithm of the median of the ratio of collector cutoff current at 1, 2, 3, 5 and 7.5 thousands of hours of operation to initial collector cutoff current. The accelerating stresses employed were ambient temperature and power dissipation. Samples ranging in size from 17 to 20 were placed on test at each stress level.

SUMMARY OF RESULTS

This accelerated test method is based on the assumption that degradation of transistors due to temperature effects occurs in accordance with the Arrhenius Model. The Arrhenius Model as applied to this part states that the following is the formula for converting accelerated test data to equivalent time at rated stress:

$$t = \left[e^{-B\left(\frac{1}{T'} - \frac{1}{T}\right)} \right] t' \qquad (3.16.1)$$

where t = mean life at rated temperature

t' = mean life at accelerated temperature

B = an empirical constant

T' = accelerated temperature in °K

T = rated temperature in °K

The following steps represent a numerical example describing the use of this model in the accelerated life testing of this transistor type:

1. The linear degradation with time is the rate of change of the logarithm of the median of the ratio of collector cutoff current at time t to the same measurement at time t=0. Between 17 and 20 parts were put on test at the following conditions:

Condition Designation	Operating Conditions	Power Dissipation
	°C	mw
B	25	150
C	150	83
E	25	500

   Two groups of parts were also tested at zero power. They are not mentioned here since the test results indicated different acceleration factors would probably be required.

2. The slopes of the degradation versus time lines were calculated. For condition designations B, C, and E the slopes were calculated as .00465, .03047 and .02045 respectively.

3. The slopes were plotted on a graph with degradation rate on the ordinate and the reciprocal of junction temperature in degrees Kelvin on the abscissa. A line of best fit was calculated through the three points by the method of least squares. This line had a slope of 1096. The junction temperature was calculated by assuming a coefficient of thermal resistance of .23 which is the value which best lines up the three points on the least squares plot.

4. The acceleration factor is then calculated by the formula

$$\tau = 10^{-1096\left[\frac{1}{T'} - \frac{1}{T}\right]} \qquad (3.16.2)$$

where τ = the acceleration factor

T' = accelerated ambient temperature in °Kelvin

T = rated ambient temperature in °Kelvin

Note: The above formula holds for storage tests. If the tests are performed with power on the parts, then the T and T' in the above formula must be replaced by the junction temperatures which are designated as S and S'. The formula for S is:

$$S = T + \alpha P \qquad (3.16.3)$$

where α = coefficient of thermal resistance

P = power dissipation

T = case temperature

Likewise for S' the calculation is

$$S' = T' + \alpha P \qquad (3.16.4)$$

Therefore, if 25°C, 150 mw is represented as rated operating conditions, the calculation of the acceleration factor for 25°C, 500 mw is

$$\tau = 10^{-1096\left[\frac{1}{413.2} - \frac{1}{332.7}\right]}$$

$$\tau = 4.40$$

Then if one tested a group of these parts to failure at accelerated (i.e.: 25°C, 500 mw) stress levels and the mean failure time was 1000 hours, then by the formula:

$$t = \tau t' \qquad (3.16.5)$$

$$t = 4.40\ (100)$$

$$t = 4400$$

The estimate of mean life at rated stress levels (i.e.: 25°C, 150 mw) would be 4400 hours.

In addition to testing the parts to failure at the accelerated stress level, the failures experienced should be predominantly due to drift of the collector cutoff current outside its stated specification.

INSTRUCTIONS FOR USE

The following is a step by step description of how the subject accelerated life testing method should be applied:

1. Place n transistors of the specified type on test at m different temperature levels. Measure the collector cutoff current (I_{cbo}) initially and at 1,2,3,5,7.5 thousands of hours of operation. At each measurement interval calculate the ratio of the $I_{cbo,t}/I_{cbo,0}$ for each part on test. Rank the n ratios and

select the median. Take the logarithm of the median at each time interval.

2. Plot the logarithm of the median of the ratio $I_{cbo,t}/I_{cbo,0}$ versus time for each temperature stress level. Calculate the line of best fit for each stress level by the method of least squares. The slope of each of these lines represents the degradation rate over time due to each temperature level.

3. Make an Arrhenius Plot by preparing a graph with the logarithms of the slopes of Step 2 as the ordinate and the reciprocal of the absolute temperature as the abscissa.

4. Determine if true acceleration exists by fitting a straight line to the points on the Arrhenius Plot. If there is a reasonably good fit of the observed points to the straight line, it can be assumed that the Arrhenius Model represents the degradation with time of the logarithm of the median of the collector cutoff current ratio. The slope of the regression line is B of forumula 3.16.1.

5. The acceleration factors are then calculated by formula 3.16.2 if the accelerating stresses are ambient temperatures. If the tests are powered the junction temperatures are calculated from formulae 3.16.3 and 3.16.4 and these values are used in formula 3.16.2 for calculating acceleration factors.

6. The acceleration factors are then used with the mean time to failure from an accelerated life test in formula 3.16.5 to calculate an estimate of mean part life at rated stresses.

LIMITATIONS/RANGE OF APPLICABILITY

The determination as to whether true acceleration exists is dependent upon a straight line fitting the points on the Arrhenius Plot. In the data used to develop the use of this method on the subject transistors only three points were available for this determination. This is a weakness that must be corrected before it can be conclusively proven that the Arrhenius Model is appropriate. A second limitation is the manner in which the junction temperature was calculated. Reference 321 says "Because the coefficient of thermal resistance was not accurately known for these transistors, that value of α was chosen which would most nearly line up the points on the Arrhenius Plot." This statement indicates that the junction temperatures were calculated using a favorably assumed rather than a measured coefficient of thermal resistance.

The Arrhenius Model's use carries with it the requirement that the distribution of failure times at both accelerated and at rated stress levels must be from the same distribution family.

With regard to range of applicability, the manufacturer of the parts used in the development of this test method was not given. Hence, it is not known whether the method can be extended to parts of other manufacturers. There is evidence, however, that the method cannot be extended to other transistor types without evaluating other degradation parameters. This is based on the report in Reference 321 that the Arrhenius Model has been applied to reverse leakage current on 2N705 transistors and to the failure rate of 2N559 transistors. From this, it would appear that different transistors might exhibit different degradation parameters which agree with the Arrhenius theory.

REFERENCES

320 and 321.

ALT Evaluation	
Assumptions Generally Accepted	___
Empirical Proof	X
Algorithm	X
Physical Model	X

3.17 PART NAME AND DESCRIPTION: All Semiconductors, Film Resistors, Capacitors, and Film Deposited Networks.

This ALT method is known as step stress testing and has been tried on a large variety of parts with an equally large variety of results. To demonstrate its use, a single set of results has been selected as being representative and they are presented here as a guideline. The parts selected as an example are 2N559 pnp diffused base germanium mesa transistors which are intended for low power, high speed switching. They were vacuum baked and back filled with a dry gas before encapsulation.

SOURCE: Step stress testing was developed by G. A. Dodson and B. T. Howard of Bell Telephone Laboratories. The methodology is based on work by Marcel Prot in fatigue testing. Many others have contributed heavily to the present state of the art of step stress testing and these contributions are presented in the section on References at the end of this subsection.

PURPOSE OF TEST

Since this ALT method has been used on such a wide spectrum of parts, there has also been a number of purposes for its use. Dodson and Howard used the method for calculating the relationship between the reciprocal of storage temperature in degrees Kelvin and the logarithm of median failure time. Others have used it for comparing the rate of change of various operating parameters with time and temperature and operating stresses while many have used it for qualitative comparisons of new products, new specifications or to measure the uniformity of quality levels.

A further use for step stress testing which has proved most valuable is in the selection of the proper stress levels to be used in constant stress tests in order to obtain both the desired failure mode and the assurance of a sufficient number of failures in a reasonable length of time.

DEGREE OF VALIDATION

It is quite difficult to evaluate this particular ALT method because the literature displays so many varying opinions on its merits and so many variations on how it has been applied. In general, however, it appears to merit consideration as a qualitative method of comparing resistance to stress and shows promise of being developed into a useful technique for calculating numerical reliability characteristics. The question of its efficiency as an improvement over constant stress testing is not clear. While it is true that it reduces the number of both early and

late failures, the problems of applying and reapplying the accelerating stresses accurately largely overshadow this advantage.

A change of slope of the plot of the reciprocal of absolute temperature versus some function of failure time usually heralds a change in the mode of failure observed. Ample evidence of this relationship proves the utility of step stress testing in the determination of the stress level at which failure modes will shift. No single piece of literature contained a sufficient amount of repetitions of the use of the method on similar parts with like results to claim full validation. On the other hand, several authors had poor agreement with established results Consequently, it must be viewed that the method has merit but more proof is required of its validation.

DESCRIPTION OF TEST METHOD

A sample of unspecified size of 2N559 transistors was randomly selected from a production run and divided into several test groups. The first group was stressed by placing it in a furnace at a given temperature for a fixed time interval. At the end of the time interval, the parts were removed from the furnace, cooled to room temperature and tested for the operating parameters of interest. The number of failures was recorded, the parts were returned to the furnace and the temperature was raised to a higher level. They were kept at the higher stress level for exactly the same time as the previous test run. This same procedure was followed until all of the parts in a test group had failed. The procedure was repeated on the other groups of parts using different time intervals. In the experiment described here the time intervals at each stress level for each group of parts varied from 20 minutes to one week. The magnitude of the increase in temperature at each step was made equal in terms of $1/T°K$.

The definition of failure was based on the following electrical specifications:

Collector Breakdown Voltage (at 100 μa): $BV_C \leq 12V$

Collector Reverse Current (at -5V): $I_{CO} \leq 5\ \mu a$

Emitter Breakdown Voltage (at 100 μa): $BV_E \leq 3.2V$

SUMMARY OF RESULTS

The method of step stress testing is based on several important assumptions:

1. The S-N curve can be transformed into a straight line by some mathematical transformation representing a physical model. In the example presented here the transformation is logarithmic on the time scale and the reciprocal of the absolute temperature on the stress scale.

2. The failure times at any point on the S-N curve must be normally distributed with equal variance.

3. The transformed S-N curve represents the effects on life of a single failure mechanism or at least on a very dominant failure mechanism.

4. The variation in the failure times at any given stress level is due only to differences in the device under test.

5. The probability of failure at any given point in the stress-time domain is independent of how a part arrives at that point.

6. The identical S-N curve can be obtained either by constant stress tests at several different time intervals where it is assumed that time is held constant as the stress is increased. Figure 3.17.1 is a schematic diagram of this assumption. A_1 and A_2 represent failure distributions derived by constant stress accelerated tests. A_3 represents the failure distribution at rated stress levels. Distribution curves B_1, B_2 and B_3 represent failure distributions with fixed time intervals and varying stresses. In the diagram it is assumed that the mean of distribution A_2 and the median of distribution B_3 are equal and hence the results of either constant stress tests or step stress tests give equivalent results.

INSTRUCTIONS FOR USE

The following is a step by step instruction for using step stress testing for storage temperature testing of 2N559 transistors.

1. Select several time intervals for use in the step stress program. In the example on 2N559 transistors the intervals selected were 20, 120, 1440 and 10,080 minutes.

2. Select temperature intervals such that 1/T°K are of equal width.

3. Place a randomly selected group of parts in an oven at a moderate temperature for 20 minutes. At the end of this time period remove the parts, cool them to room temperature and measure the operating parameters of interest. For the 2N559 transistors the parameters measured were collector breakdown voltage, collector reverse current and emitter breakdown voltage.

4. Record the number of failed parts, put the parts back into the furnace, raise the temperature to the next higher stress level, and hold for 20 minutes.

5. Repeat the cooling, measuring, recording of failures procedure until all parts tested at the 20 minute time interval have failed. Then repeat the procedure with the parts at 120, 1440 and 10,080 minute time intervals.

6. Plot the 20 minute time interval failure data on normal probability paper with the reciprocal of the absolute temperature on the ordinate as in Figure 3.17.2. If a straight line fits the plotted points, the assumption regarding normality is fulfilled. Repeat the plot for the other time intervals.

7. Prepare an S-N curve with logarithm of the time interval on the abscissa and the reciprocal of the absolute temperature on the ordinate. Plot the temperature at which the median (or any other desired percentile) of each sample group failed and the logarithm of the time interval of the steps. An example of the S-N curve for 2N559 transistors is shown as Figure 3.17.3. The S-N curve is then extrapolated to lower temperatures to predict the expected life at rated temperatures.

LIMITATIONS/RANGE OF APPLICABILITY

The majority of users of step stress testing make their determination as to whether or not true acceleration exists (the S-N curve can be transformed to a straight line) based on perhaps three different step interval times. This means that the line of best fit through the points on the S-N curve is calculated on a sparse amount of data. Therefore, the assumption that step stress testing applies as a predictor of life is often based on an insufficient number of data points.

The assumption that time can be held constant while the stress is increased through several cycles of temperature stress sometimes covering a period of weeks seems incompatible with most theories of cumulative damage.

The range of applicability of step stress testing is quite wide as seen from the available references. It has been applied with varying degrees of success to nearly all types of semiconductor parts and several varieties of resistors and capacitors. Its main area of application has been in comparison tests and in the selection of the proper levels for use in constant stress tests and in determining stress levels where changes in failure mode occur.

REFERENCES

The following references represent the various aspects and part types to which step stress testing has been applied:

Resistors: 32, 33, 34, 36, 54 and 174.
Capacitors: 32, 33, 34, 36, 45, 111 and 174.
Diodes: 32, 33, 34, 36, 54, 97, 129, 137, 138, 174 and 280.
Transistors: 32, 33, 34, 36, 54, 85, 95, 96, 97, 129, 136, 138, 144, 145, 149, 174, 207, 245, 265 and 280.
Semiconductor Networks: 5.
Miscellaneous: 228, 272, 305 and 328.

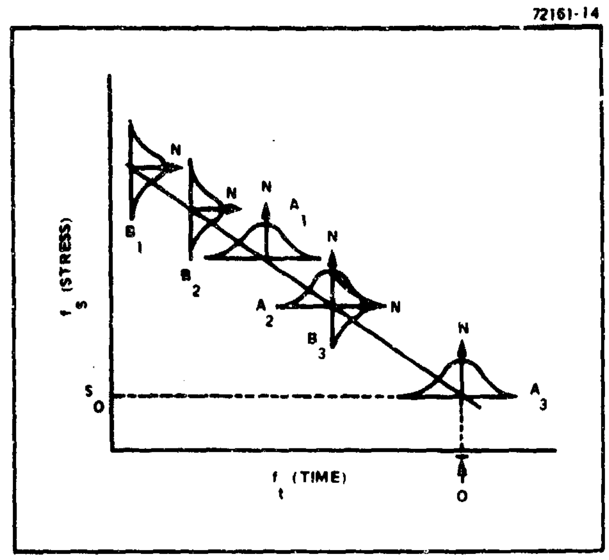

Figure 3.17.1. Linear Accelerated Aging Curve

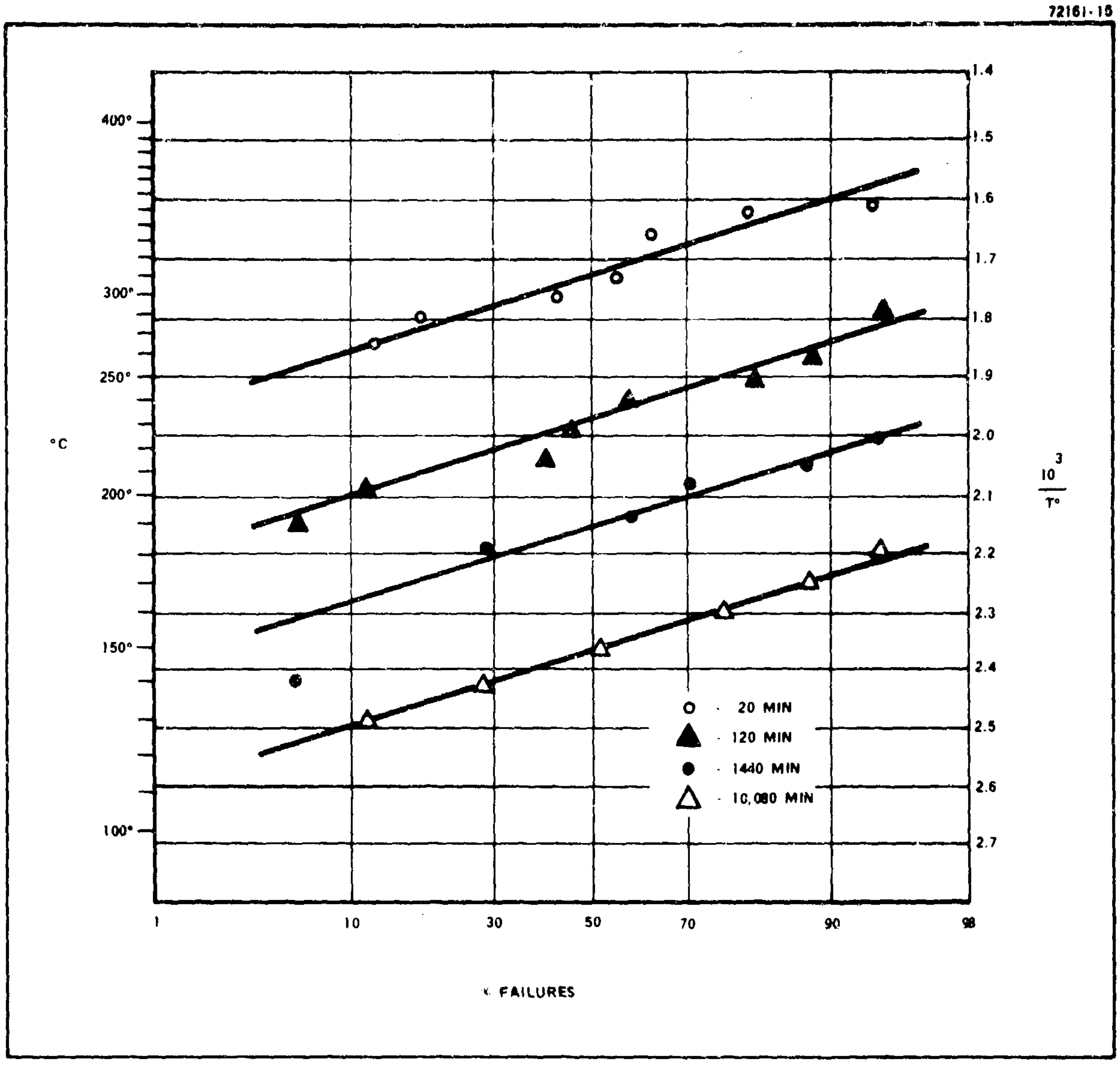

Figure 3.17.2. Failure Distribution of 2N559 Vacuum Baked Transistors

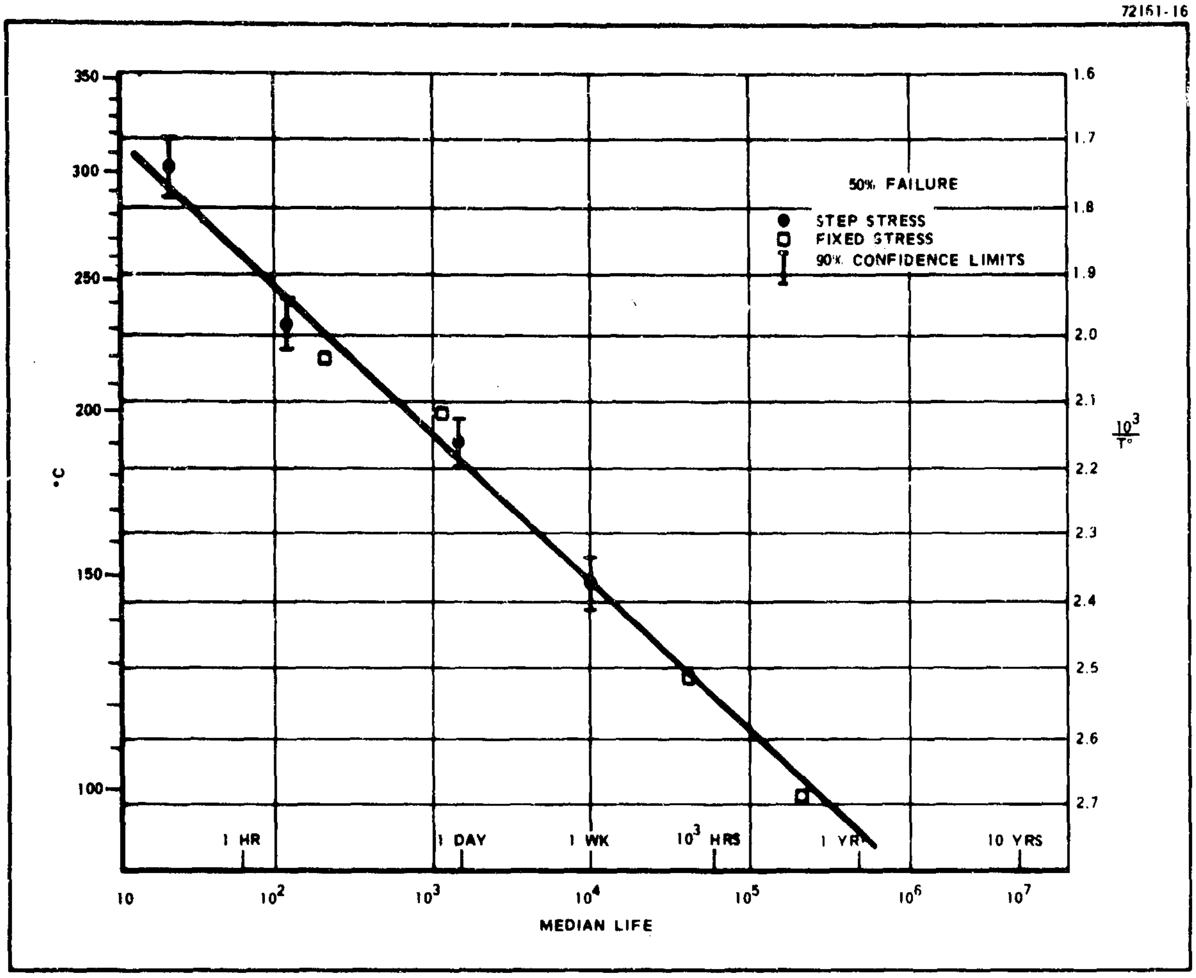

Figure 3.17.3. Acceleration Curve for 2N559 Vacuum Baked Transistors

4.0 ALT METHODS FOR MECHANICAL PARTS

This section of the handbook is similar to Section 3 except that it presents the significant contributions to the state of the art of accelerated life testing of mechanical parts. The methodology for selecting parts for inclusion in this section as well as the rules for evaluation of the degree of validation of each are the same as those used in Section 3.

4.1 PART NAME: Ball and Roller Bearings of Various Types, Materials, and Lubrication Materials.

ALT Evaluation	
Assumptions Generally Accepted	X
Empirical Proof	X
Algorithm	X
Physical Model	

SOURCE: Numerous Studies and Reports of the Bearing Manufacturers, American Standards Association, and Independent Agencies, Both Government and Private.

DEGREE OF VALIDATION: Ample evidence is available in the references for validation of this ALT, particularly in No. 69 "Electromechanical Component Reliability" by George Chernowitz; No. 218 "Statistical Investigation of the Fatigue Life of Deep-Groove Ball Bearings" by J. Lieblein and M. Zelen; and No. 494 "B3-11-1959 American Standard Method of Evaluating Load Ratings for Ball and Roller Bearings." There are slight differences in the value of the exponent in the algorithm depending on bearing type and manufacturer. However, the industry has generally standardized on the use of 3. Tests have not been performed to verify that the subject life rating formula is applicable for large diameter bearings.

PURPOSE OF TEST: To Predict the Life of Ball or Roller Bearings at Specified Conditions of Load Other Than That for the Rating Life.

DESCRIPTION OF TEST METHOD

1. Definition of Failure:

   The point at which the first evidence of fatigue develops in the material of either ring or of any of the rolling elements. It has been shown that if bearings are installed correctly, failure will be from fatigue and the failure times will be according to the Weibull distribution.

2. Method of Gathering Data:

   The specimen worksheet in Figure 4.1.1 presents the basic bearing data required for a life test. The worksheet is taken from Reference 218 and presents a bearing manufacturer's data for an actual test, together with the determination of the Weibull slope and the B-10 life.

3. Method of Applying Accelerating Stress:

   The accelerating stress is imposed on the bearing by the application of radial load at a fixed speed (rpm of the bearing inner race in most cases).

SUMMARY OF RESULTS

1. Mathematical Models

Descriptive Definitions (from Reference 494):

Life. The "life" of an individual ball or roller bearing is defined as the number of revolutions (or hours at some given constant speed) which the bearing runs before the first evidence of fatigue develops in the material of either ring or of any of the rolling elements.

Rating Life. The "rating life" of a group of apparently identical ball or roller bearings is defined as the number of revolutions (or hours at some given constant speed) that 90 percent of a group of bearings will complete or exceed before the first evidence of fatigue develops. As presently determined for ball or roller bearings, this rating life is approximately one-fifth of the life which 50 percent of the group of bearings will complete or exceed.

Basic Load Rating. The "basic load rating" is that constant stationary radial load which a group of apparently identical ball bearings with stationary outer ring can endure for a rating life of one million revolutions of the inner ring. In single-row angular-contact ball bearings, the basic load rating relat to the radial component of the load, which results in a purely radial displacement of the bearing rings in relation to each other.

Load Ratings for Specific Speeds. Load ratings, if given for specific speeds, are to be based on a rating life of 500 hours.

Equivalent Load. The "equivalent load" is defined as that constant stationary radial load, which, if applied to a bearing with rotating inner ring and stationary outer ring, would give the same life as that which the bearing will attain under the actual conditions of load and rotation.

Nomenclature:

L_n = Life in Millions of Cycles for 100% Survival.

C = Basic Load Rating.

P = Equivalent Load or Load (depending on type of bearing -- deep groove ball bearing, roller bearing, thrust ball, bearing, etc.).

Reference No.: ____________

Bearing Mfg. by: ____________

Bearing Tested by: ____________

Date of Test: 8/26/46

Bearing No.: ____________

Load: 580 R.L.

Speed: 2000 r.p.m.

Lubrication: Type: Jet Oil

Frequency: ____________

Ball No. and Dia.: ____________

Contact Angle: ____________

Groove Radius: Inner Ring: 51.6%

Outer Ring: 53.0%

Number of Rows: 1

Bore: 20 mm.

O.D.: 42 mm.

Lot Size: 25 Taken on 23

Bearing temperature measured on outer ring at point of maximum load ____________

Material: Type: ____________

Source: ____________

Rockwell Hardness of: ____________

Inner Ring: 63.5

Outer Ring: 64.0

Balls: ____________

Ball Failure: 13 52%

Inner Ring Failure: 5 20%

Outer Ring Failure: 1 4%

Test life in 10^6 revolutions:

Median: 68.0

Mean: 71.0

B-10: 29.0

Slope of Curve: 2.23

Test No.: 3183

Lot: 71

Table Ordered According to Endurance Life

Brg. No.	Endurance Mill. Revs.	Type of Failure	Remarks
16	17.88	Ball	
10	28.92	Ball	
5	33.00	Ball	
19	41.52	I.R.	
9	42.12	Ball	
11	45.60	Ball	
15	48.48	Ball	
12	51.84	Ball	
20	51.96	Ball	
18	54.12	I.R.	
13	55.56	I.R.	
1	67.80	Ball	
2	67.60	L. Bore	Omitted
3	67.80	L. Bore	Omitted
4	68.64	Ball	
6	68.64	L. Bore	
25	68.88	Disc.	
22	84.12	Ball	
17	93.12	Ball	
7	98.64	I.R.	
23	105.12	I.R.	
24	105.84	Disc.	
21	127.92	Ball	
8	128.04	O.R.	
14	173.40	Disc.	

Figure 4.1.1 SPECIMEN WORKSHEET

Model:

$$L_n = \left(\frac{C}{P}\right)^p \text{ or } \frac{P_1}{P_2} = \left(\frac{L_2}{L_1}\right)^{1/p} \qquad (4.1.1)$$

Where p is 3 for deep groove annular ball bearings under the application of radial loads.

2. Analysis Methods

Failures in bearings occur in only one way -- fatigue; if the bearings are properly installed.

Failures in a group of bearings tested under the same load and speed conditions are in accordance with the Weibull distribution (see Limitations/Range of Application).

Figure 4.1.2 shows the good correlation obtained for life tests on two types of bearings compared with theoretical curve for the model, $L_n = \left(\frac{C}{P}\right)^3$.

INSTRUCTIONS FOR USE

Manufacturer's catalog data are available for the rating life and the basic load rating for the make and configuration of bearing desired.

To find the life for accelerating conditions, solve the life equation for the accelerating load. (The bearing speed should be the same as that used by the manufacturer.)

Verify the calculation by testing a sample group of bearings at the expected load. A Weibull plot of the failures should be made to verify that the failures indeed are of Weibull distribution. If the distribution is not Weibull, examine the bearing installation and the test equipment to insure the installation is correct and that the load is applied properly. If there is difficulty with either of these two items, fatigue may not be the mode of failure.

LIMITATIONS/RANGE OF APPLICATION

1. The value of 3 for the exponent p in the model is an agreed-upon approximation of the bearing industry and was established by the American Standards Association, Reference 494, for which J. Lieblein and M. Zelen performed the statistical analyses, Reference 218. The analyses show that data collected and submitted by four bearing manufacturers from their files, going back many years, produced small differences in the value of p from company-to-company. It is not known if data from the companies' current files would be closer in agreement, but hopefully closer agreement would be found, because of increasing standardization of measuring instruments and techniques; sizing, selection, and assembly of the bearing components;

materials; lubrication; installations; and other factors relative to bearing production and application.

2. This ALT is applicable to deep-groove annular ball and roller bearings, except that the value for the exponent p becomes $\frac{10}{3}$ for roller bearings. In addition, the value of C, the rating life, versus P, the equivalent load, is dependent upon the bearing configuration: the ball or roller diameter, number of rows, number of balls and rollers, contact angle, material, radius of ring raceways, effective roller length, and other factors. Therefore, great care must be used to select these values from the correct tables supplied by the manufacturer.

3. Recent work at SKF (see Reference 153) indicates that the Weibull distribution curve is accurate only in the failure range from 7% to 60%. Below and above this range, bearing fatigue lives are longer than those predicted by the Weibull distribution.

4. For large diameter bearings not enough parts have been tested to verify if the life rating formula holds.

REFERENCES

59, 65, 69, 71, 153, 218, 350, 469 and 494.

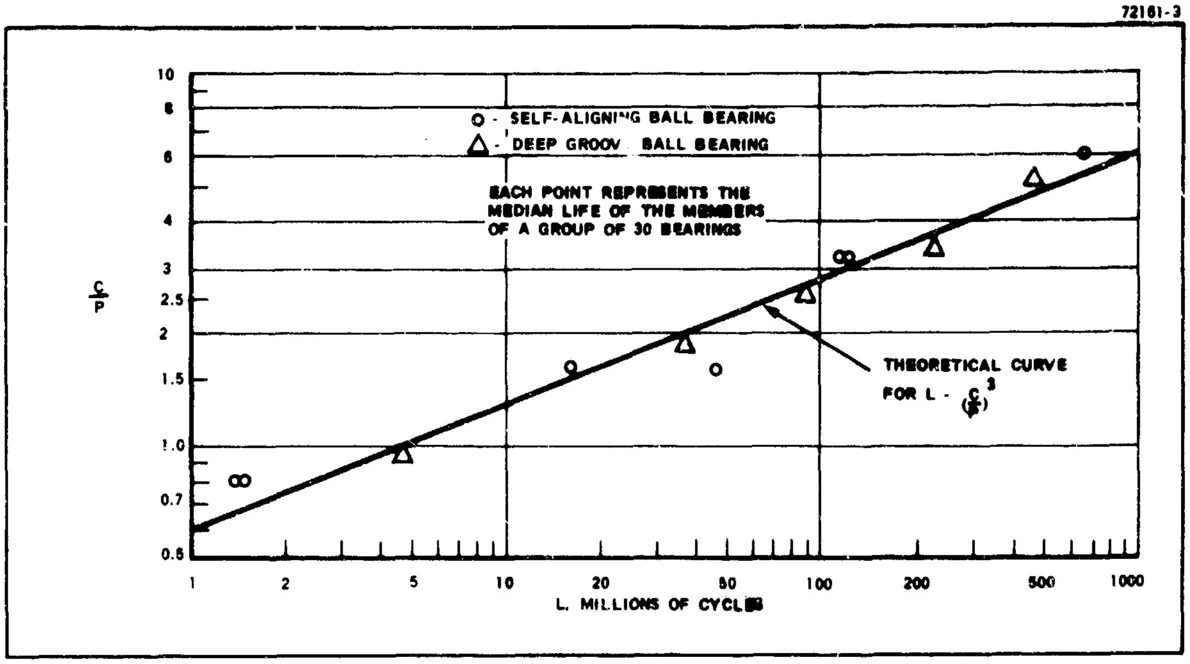

Figure 4.1.2. Bearing Life (L) in millions of Cycles, L, as a Function of the Ratio C/P

4.2 PART NAME: Gears of Various Types, Materials, and Lubricants.

SOURCE: A Number of Studies on Reliability Prediction for Mechanical and Electromechanical Parts.

ALT Evaluation	
Assumptions Generally Accepted	X
Empirical Proof	___
Algorithm	X
Physical Model	___

DEGREE OF VALIDATION: Validation evidence for this ALT is meager. The references and supporting material for the references contain some verifying data. However, the user must analyze each unique configuration and application and verify the results by tests and statistical analysis.

PURPOSE OF TEST: To Predict the Mean Wear Life of Gears at Specified Conditions of Load.

DESCRIPTION OF TEST METHOD

1. Definition of Failure:

The point at which backlash becomes excessive between the pinion and the driven gear, caused by loss of material from the teeth faces through fatigue wear. The failure is gradual, characterized by increasing backlash with running time until the backlash exceeds that allowable by the gear train performance specification. (In the case of bearings, the fatigue wear failure of the balls and races occurs suddenly; that is, bearing failure progresses extremely fast after the first indication of wear.)

2. Method of Applying Accelerating Stress:

Considering a pair of spur gears (a pinion and a driven gear) operated at constant speed by a motor attached to the pinion shaft, the accelerating stress is imposed by application of load in the form of torque to the driven gear shaft or output shaft. The device for load application can be a prony brake, fluid dynamometer, electric dynamometer or other mechanism.

SUMMARY OF RESULTS

1. Mathematical Models

Nomenclature:

W_w = Wear Load, the maximum permissible load above which wear of the tooth surface would be excessive - empirically established from allowable load.

d = Pitch Diameter.

F = Tooth Face Width.

k = Factor Depending on Materials in Contact.

N_p = Number of Teeth on Pinion.

N_g = Number of Teeth on Gear.

C = The Specific Dynamic Capacity (90% survival).

K = Gear Load Stress Factor.

K_1 = Load Stress Factor Obtained for Roller Mating Materials.

ϕ = Gear Contact Pressure Angle.

L = Life in Millions of Tooth Contact Cycles for 100% Survival.

P = Equivalent Load or Load, Depending on Type of Gear (spur, helical, bevel, etc.).

Basic Relationships:

Wear Load, W_w

$$W_w = Fkd\left(\frac{2N_g}{N_p + N_g}\right) \tag{4.2.1}$$

Gear Load-Stress Factor

$$K = \frac{K_1 \sin \phi}{4} \tag{4.2.2}$$

Mathematical Model

$$L = \left(\frac{C}{P}\right)^x \tag{4.2.3}$$

, which is the same as the life equation for bearings.

or $$\frac{L_1}{L_2} = \left(\frac{P_2}{P_1}\right)^{1/x} \tag{4.2.4}$$

2. Analysis Methods

As described above, failures in properly designed and installed gears occur mainly through rolling contact surface fatigue at the faces of the teeth.

Failure times in a group of identical gears are in accordance with the Weibull Distribution.

Little data on gear lives exists, possibly because of the very large number of combinations of gear sizes, ratios, tooth forms, pitches, materials and combinations of materials, lubricants, applications, and environments as opposed to the relatively small numbers of gears produced for each design configuration.

It is necessary to make approximations of life calculations, utilizing Equations (4.2.1) or (4.2.2) and the Model (4.2.3) (estimating the value of the exponent x) for a particular gear configuration and application. Then a large number of verification tests must be run and the results analyzed by statistical techniques to firmly establish the exponent x. Reference #218 describes the mathematics for establishing the life equation exponent value for deep-groove ball bearings and may be used as a guide for establishing the gear life equation exponent.

The application may not permit the use of catalog gears from a manufacturer, and in this case the gears must be specially designed by standard gear design textbook techniques by the user or the manufacturer. In this case, the design must include consideration of the wear load, gear-load stress factors, and life.

INSTRUCTIONS FOR USE

1. For simplicity, assume that for a specific gear train application, the basic design has been established such that gears may be chosen from a manufacturer's catalog; for instance Boston Gear, which gives ample information for selecting catalog gears without the necessity of resorting to elaborate mathematics.

2. The manufacturers do not give catalog gear fatigue data; therefore, calculate the wear load W_w, by Equation (4.2.1) above, which is an approximation of the allowable load. (Load-stress factors may also be used to calculate the wear load, in which case use Equation (4.2.2). The use of load-stress factors permits the calculation of the limiting wear load W_w.)

3. The allowable load, equivalent to the fatigue limit, represents a point on the fatigue life curve (see Figure (4.2.1).

4. The specific dynamic capacity C, represents an estimate of the fatigue limit, or is a point above the fatigue limit (on the S-N diagram) which is on the 90% survival curve.

5. Substitute the wear load W_w in the Model (4.2.3) for C, the specific dynamic capacity.

6. Estimate the model exponent x.

7. Solve the model for the life L.

8. Verify the results by tests and statistical analysis to firmly establish the model exponent x.

9. Gear lives for wear loads other than the design load can be calculated from the Model (4.2.4) or taken from the curves constructed for statistical analysis, and the results verified by a small amount of accelerated life testing.

LIMITATIONS/RANGE OF APPLICATION

1. The cost of running tests for a group of gears to firmly establish the life equation exponent x for the model must be added to the cost of the ALT. Therefore, the use of this ALT is limited to only those situations where the added costs are warranted.

2. Availability of data for k and K_1 in Equations (4.2.1) and (4.2.2) appears limited. Perhaps, since these equations are used for estimation of wear load W_w, similar data from bearing manufacturers would be suitable for use in the equations.

3. Gear wear life data from manufacturers has not been found during this study. It is believed that thus far, because of the numbers of possible gear and gear train design configurations and applications, the manufacturers have found it is not economical to pursue a course to obtain this data for their catalog items.

REFERENCES

59, 69, and 71.

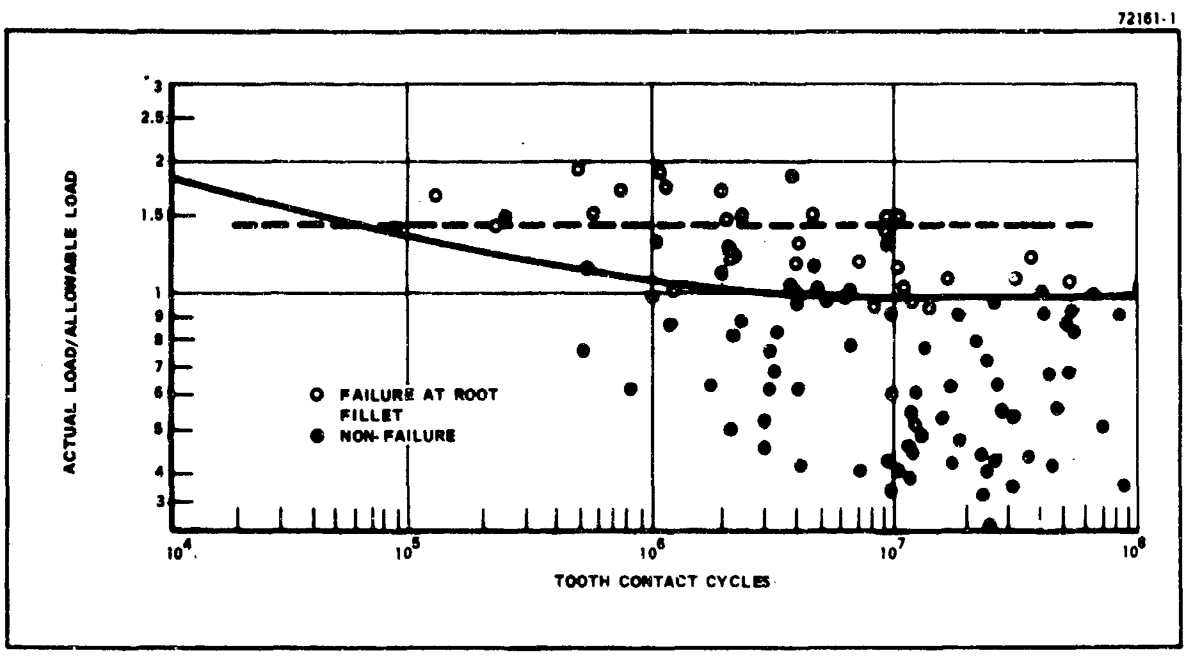

Figure 4.2.1. Ratio of Actual to Allowable Load vs Tooth Contact Cycle

4.3 PART NAME: Gears, Fatigue Stress by "Measured Weakening" Technique.

ALT Evaluation	
Assumptions Generally Accepted	X
Empirical Proof	
Algorithm	X
Physical Model	

SOURCE: AD 617 567 "Research on Accelerated Reliability Testing Methods Applicable To Non-electronic Components of Flight Control Systems," W. F. Johnson, et al, March 1965, Reference 181.

DEGREE OF VALIDATION: Each of the test phases required to develop the acceleration relations has been validated theoretically, according to the authors, and proven feasible separately, but not repeated sequentially to obtain specific combined values because of program limitations. Development of specific life predictions is dependent upon this sequential development of combined values.

PURPOSE OF TEST: To Predict the Fatigue Stress Life of the J85-5 Turbojet Engine Afterburner Exhaust Nozzle Actuation System Power Unit Output Gear.

DESCRIPTION OF TEST METHOD

1. Definition of Failure:

   Fracture of gear tooth at or near the root caused by fatigue stress.

2. Method of Applying Accelerating Stress:

   Note: The test program was carried out in three phases; a. system, b. part, and c. material.

   a. System

   The mission profile was analyzed and found that an increased duty cycle could be formulated such that the one hour mission could be simulated by a five minute test (acceleration factor, F_1, of 12 to 1). 539 accelerated missions were run.

   The output gear tooth roots were pre-cracked or "measurably weakened" by a specially developed capacitor discharge spot welding method such that an .030" pre-crack was formed at each end of each fillet on each side of each tooth (see Figure 4.3.1).

b. Parts

A combination of an electrodynamic shaker system for the vibratory load and a relatively soft spring shock cord for the steady state load were used to apply the combined loads to one tooth on one gear held in a special test fixture.

A small spot weld mark in the corners of the tooth root was used for a pre-crack (smaller than .030"). A .030" crack developed after 32,000 cycles. A curve for crack propagation was developed by subsequent regular interval inspections. Several gears were used to establish an S-N curve.

c. Materials

The materials test phase was used to establish the material constants necessary to develop the acceleration factors to relate fatigue life to crack propagation. Five R. R. Moore test specimens were run at different constant stress levels to complete failure at 10,000 cycles per minute. From this load spectrum 140,000 psi was selected to generate cracks in the other specimens in a short time (13 minutes). An optimum crack length was also determined. It was found that a pre-crack length of 0.200" should have been used for the System Test to further shorten the test.

If this crack length had been used, the system test could have been accelerated further by shortening it from 539 missions of 5 minutes each to 20 missions of 5 minutes each, or a total of 100 minutes.

SUMMARY OF RESULTS

1. Mathematical Model for Crack Propagation in the Propagation Phase.

$$\log\left(\frac{l}{l_a}\right) = ks^{\alpha} \quad (N-N_a) \qquad (4.3.1)$$

where:

l_a = starting length of crack.

l = length of crack at N cycles.

k = constant depending on material type.

s = stress load.

α = exponent of stress - depends on material, its heat treatment, the amount of steady stress, and the stress gradient.

N_a = the number of cycles associated with the crack starting length.

Equation (4.3.1) is independent of the stress concentration factor and is valid for specimens with small local notches as well as smooth specimens, provided the stress is calculated as nominal stress, and the propagation is computed after the crack tip has progressed beyond the influence of the original notch.

2. Unfortunately, in the referenced study the constraints of schedule necessitated the performance of the accelerating tests for the system, the parts, and the materials on an overlapping time scale basis; therefore, the results of the tests were not available in the proper sequence. The authors show that scheduling for future programs should place the three phases in the sequential order:

   a. Materials Tests.

   b. System Tests.

   c. Parts Tests.

   The values of the material constants μ and k should first be obtained from the materials tests and Equation (4.3.1). The optimum pre-crack length is also obtained from the material tests.

   The optimum pre-crack length is next utilized to conduct the system tests.

   Finally, the fatigue failure data obtained from the system tests is utilized to establish the parts tests.

   The parts tests are utilized to predict the gear fatigue stress life and to examine the adequacy of the design of the parts.

3. The system test phase showed that consideration of the active/inactive portions of the mission profile enabled test duration compression (or acceleration) of 12 times.

   A permissible tooth stress cycling rate in the parts tests (200-500cps) was found to be approximately 100 times that occurring in the compressed system test (approximately 10,000 cycles per hour of mission time).

   A permissible fatigue cycling rate in the materials test (10,000 on the R. R. Moore test sample) was found to be approximately 60 times that occurring in the compressed system tests.

   A possible shortening of the system test using the proper pre-crack size (determined from the materials test) - "measured weakening" - to between 5% and 40% of the normal time to failure was verified in

tests to failure of pre-cracked gears in the parts test program.

INSTRUCTIONS FOR USE

1. For a particular gear application, obtain R. R. Moore material specimens prepared in accordance with the Material Fatigue Test Specimen Specification (R. R. Moore Rotating Beam Type) given in Reference 181.

2. Perform the materials tests to obtain the S-N curve, the values of the materials constants α and k in the mathematical model (Equation 4.3.1), and the optimum pre-crack size. The optimum size is above the "non-propagating" size for the mission duty cycle and number of missions, but small enough that propagation occurs at a rate permitting regular interval halting of the test for crack measurement. "Non-propagating" is a relative term meaning that propagation rate is so slow that it does not advance significantly under 1,000,000 cycles, although it does advance.

3. Perform the system tests with the pre-cracked gear installed in the nozzle actuation power unit. The tests are performed at normal operating loads but with the time accelerated in accordance with the mission duty cycle and number of missions for the gear. The system operating time to failure will be considerably shorter than with un-cracked gears.

4. Perform the parts testing by testing a reasonable sample of pre-cracked production gears at an "equivalent vibratory stress" determined from the crack propagation rates measured under normal loads with pre-cracked gears in system tests. (To relate the test loads on the gear tooth in the parts testing to material stress, it is necessary to calculate the bending stress at the critical section of the tooth root fillet.)

5. From the data obtained in 2, 3, and 4, predict the gear fatigue stress life and determine the adequacy of the gear design for the application or the requirement for gear design changes (for which retesting is required).

LIMITATIONS/RANGE OF APPLICATIONS

1. Improvement in accuracy of measurement of cracks near the material inclusions size level.

2. Development of pre-cracking methods for other types and hardnesses of materials than those considered in this program. In other words, the technique of capacitor discharge welding to crack the gears may not be applicable to other materials, and is difficult to develop and control. Other methods of pre-cracking such as saw slots are also difficult.

3. The acceleration factors in the tests are so <u>large</u> that the fatigue <u>wear</u> conditions of the gear should also be <u>carefully</u> investigated <u>first</u> to insure that fatigue wear is not the primary mode of failure in a particular application, instead of fatigue stress.

<u>REFERENCES</u>

181.

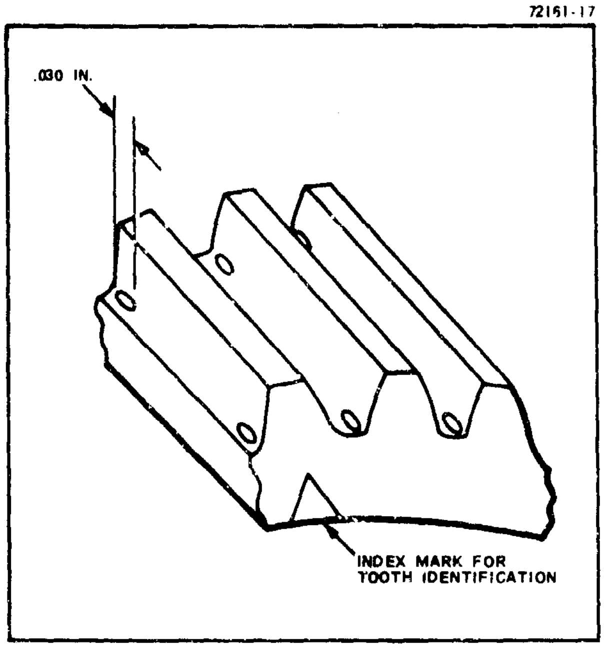

Figure 4.3.1. Section of Output Gear Showing Pre-Crack Spot Weld Marks at Max. Stress Points

4.4 PART NAME: Cast Housing, Automobile (Part Configuration, Material, and Exact Use Unspecified).

ALT Evaluation	
Assumptions Generally Accepted	X
Empirical Proof	
Algorithm	
Physical Model	

SOURCE: "Automotive Component Reliability Based on the Weibull and Log - Normal Methods" by Ned Fuller of Ford Motor Co. and Dr. Charles Lipson, Professor, Mechanical Engineering, University of Michigan, Reference 133.

DEGREE OF VALIDATION: Ample Evidence Exists in the Literature (See the Bibliography) to Validate Both the Testing and Analyses Methods Presented in This ALT as Standard, Recognized Methods in Structural Part Fatigue Stress Testing.

PURPOSE OF TEST: To Predict the Mean and B-10 Fatigue Lives and Compare Two Design Configurations of a Cast Housing, One An Improvement Over the Other.

DESCRIPTION OF TEST METHOD

1. Definition of Failure:

   Mode of failure is stress fracture through a mounting bolt hole, caused by fatigue, starting either at or adjacent to the outer surface.

2. Method of Applying Accelerating Stress:

   Two Accelerating Stresses were Applied:

   a. Time - Operational conditions were analyzed to obtain a compressed duty cycle and life. (The cast housing is loaded only in reverse operation of the vehicle, and significant stresses are imposed only under full engine output.)

   b. Load - A greater load was imposed than experienced in actual operation to reduce test time.

      The test load was alternately applied and relieved at the mounting hole boss of the cast housing while it was mounted on a plate.

3. <u>Designations of the Two Cast Housing Design Configurations:</u>

Series 1: The original design of the basic housing.

Series 2: The improved design (changes incorporated in the machining surface of the mounting boss and to increase the mounting boss thickness).

4. <u>Sample Size:</u>

Series 1: Housings - 20

Series 2: Housings - 16

Note: Two tests in each series of cast housings were terminated (suspended) without failure.

SUMMARY OF RESULTS

1. <u>Analysis Methods:</u>

a. The Series 1 and 2 test failures were plotted in Figure 4.4.1, and both showed skewed distribution conformation, which is typical of fatigue data. Therefore, it was necessary to use statistical analysis techniques that differ from those for a normal distribution.

b. For comparison purposes the data was analyzed for both the Weibull and the log-normal distributions.

c. The Weibull Distribution

The results were prepared for Weibull plotting by arranging them in the order of increasing life, and the median ranks corresponding to the sample size were assigned. The suspended items for both series were considered in the determination of the sample size and ranking.

The plotting of the test data on Weibull paper is illustrated in Figure 4.4.2. The figure also shows the 90% confidence bands established (the range of values within which the population mean life of 90% of all possible cases will fall).

The curves of Figure 4.4.2 show the smaller amount of scatter of the Series 2 data, evidenced by the greater slope of the Series 2 Weibull line.

The Series 2 median life is 8,600 cycles, with a 90% confidence range of 6,300 cycles to 10,400 cycles, as read from Figure 4.4.2.

The Series 2 B-10 life is 4,400 cycles with a 90% confidence range of 2,700 cycles to 8,000 cycles.

The Series 1 median life is 2,400 cycles, with a 90% confidence range of 1,200 cycles to 5,000 cycles.

The Series 1 B-10 life is 280 cycles, with a 90% confidence range of 64 cycles to 1,100 cycles.

d. The Log-Normal Distribution

The arrangment of the data for the Log-Normal plot is very similar to that for the Weibull procedure. The test results are tabulated in the order of increasing life and are assigned a ranking of r/(n+1), where "r" is the related test number and "n" is the total number of tests. In addition to the cycles to failure (N) and the ranking, the values for the logarithm of N are included.

The Log-Normal plots for both the Series 1 and 2 housings are shown in Figure 4.4.3.

2. Results:

a. Tabular comparison of Weibull and Log-Normal:

	Series 1	Series 2
Weibull B-10 Life	280 Cycles	4,400 Cycles
Log-Normal B-10 Life	260 Cycles	4,200 Cycles
Weibull Median Life	2,400 Cycles	8,600 Cycles
Log-Normal Median Life	2,050 Cycles	8,200 Cycles

As can be seen from the table, there is little difference in the value obtained by the two methods. The Weibull plot is a little faster, because the median ranks were available from tables, and the confidence levels are readily determinable. Advantages of the Log-Normal plot are that the confidence levels can be computed, and tests such as the "t" Test for Significance can be applied. The authors found nothing else to favor one plot over the other.

b. The improvement in the Series 2 design over that of the Series 1 is readily seen in the data of the preceding paragraph and the plots of Figures 4.4.2 and 4.4.3.

INSTRUCTIONS FOR USE

1. Select a significant number of parts, either from the field, production, or developmental, depending upon the program phase and where a suspected problem is encountered.

2. Determine the accelerating stresses that may be used for life tests by examining the operational conditions. In the present case, both time and load were available as accelerating stresses to shorten the life tests.

3. Subject the parts to fatigue tests to failure.

4. Plot the failure data to determine the conformation of the normal distribution curve as in Figure 4.4.1.

5. The normal distribution curve conformation for the fatigue mode of failure is typically skewed, therefore, analysis must be by other techniques than those required for normal distribution.

6. As the authors have shown, either the Weibull or the Log-Normal plotting method may be used to analyze the data.

7. By whichever type of plot chosen, prepare the data to obtain the median ranks, establish the 90% confidence limits and plot the data, as in either Figure 4.4.2 or Figure 4.4.3.

8. From the plot the median and B-10 lives can be read, and the determination made as to the adequacy of the part design and/or the necessity for a design change.

9. Another design configuration or another production lot can be compared with the first group of parts by selecting a significant number of the second group of parts and repeating Steps 2 through 8.

LIMITATIONS/RANGE OF APPLICATION

1. The key word in the analysis method is "significant." That is, how large a sample of the parts must be selected for the data to be statistically significant? It is obvious that the sample size should be no larger than necessary to conserve test time and facilities and analysis time, and yet the sample size must be large enough that the data is significant. The authors show two methods for determining sample size, including verification of the Log-Normal plots by the "t" Test for Significance.

2. The test and analyses methods are standard approaches, utilizable in fatigue testing of any structural part.

3. The authors do not establish S-N curves for the two groups of parts (because the situation did not require it), so that fatigue lives could be predicted for other conditions of loading and frequency of load application.

REFERENCES

133.

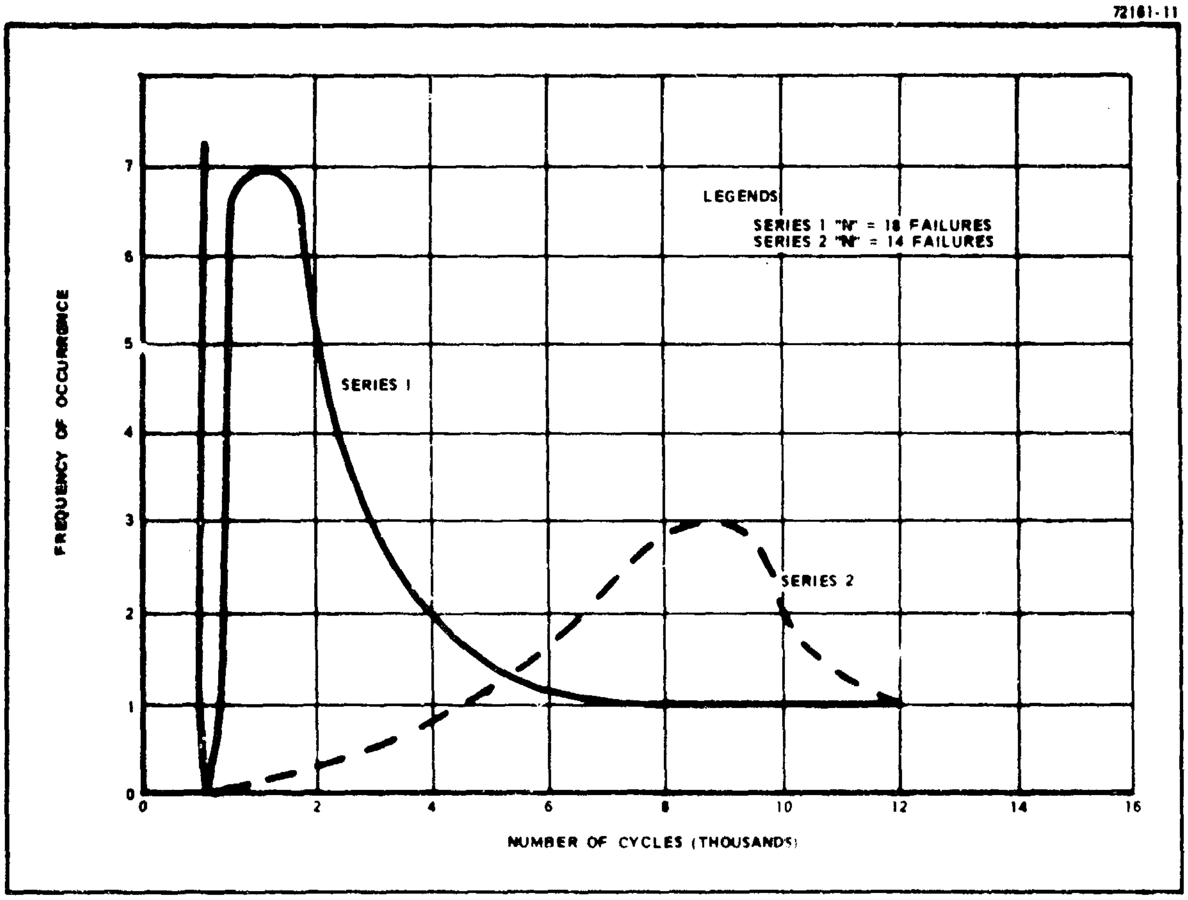

Figure 4.4.1. Distribution for Series 1 & 2

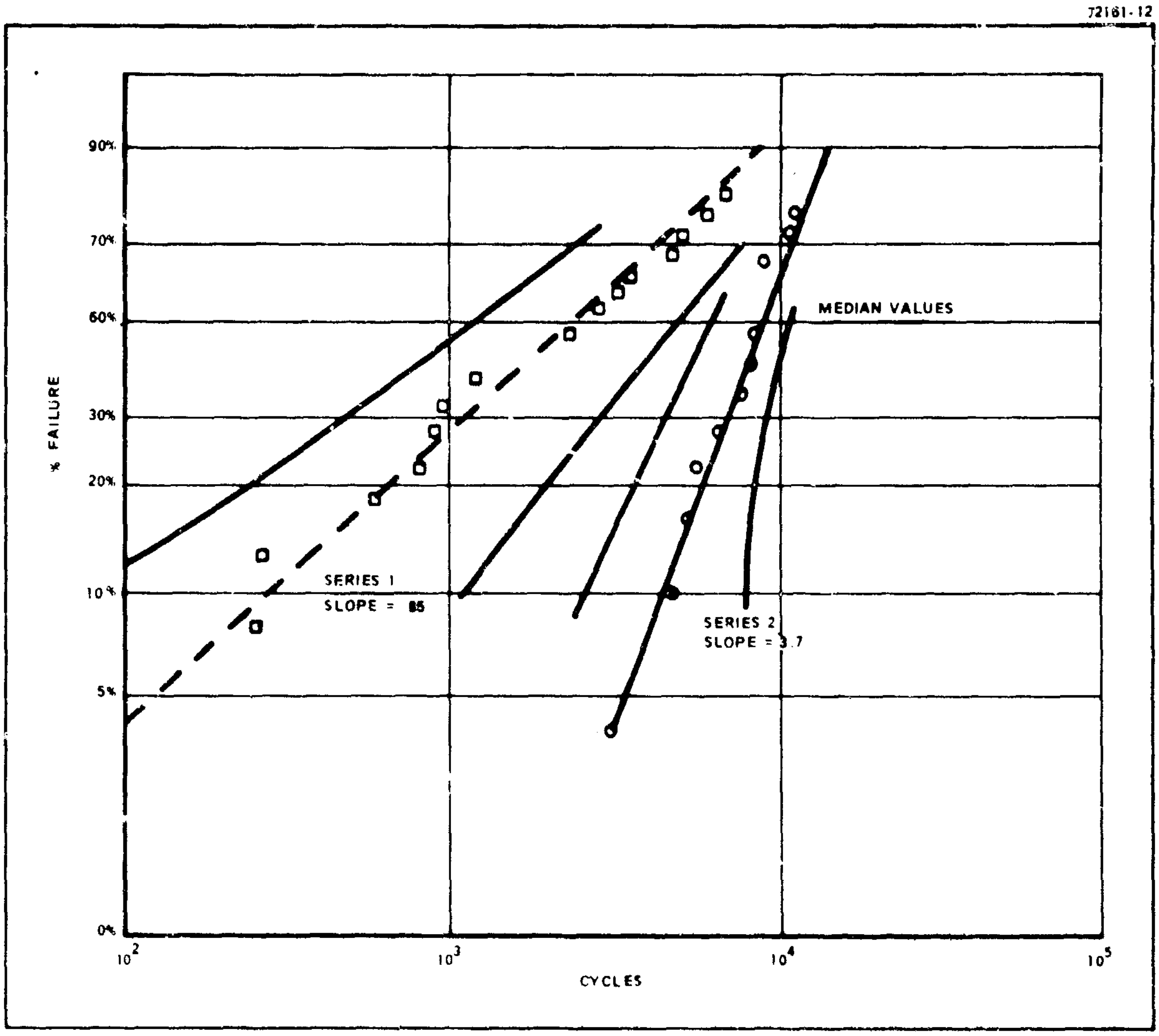

Figure 4.4.2. Weibull Plot

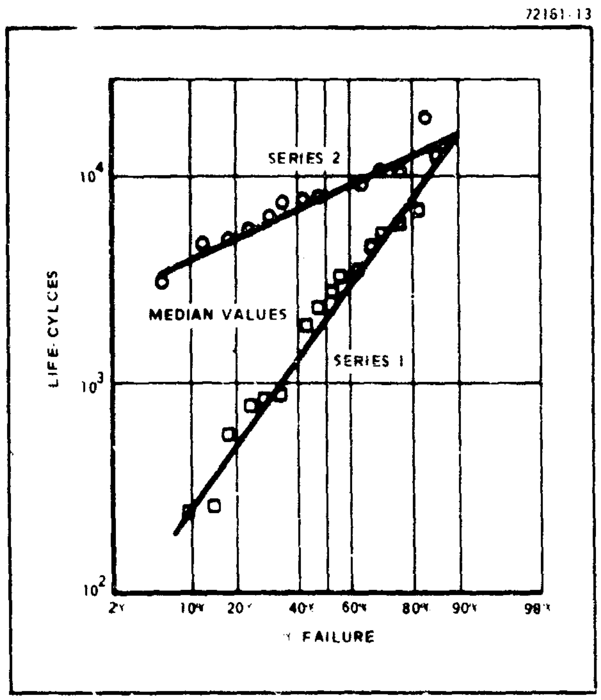

Figure 4.4.3. Log-Normal Plot

4.5 PART NAME: Structural Casing for a Turbojet Engine. (Basically a thin cylindrical shell with flange welded to each end; no material or load information specified.)

ALT Evaluation	
Assumptions Generally Accepted	X
Empirical Proof	
Algorithm	X
Physical Model	

SOURCE: "Probabilistic Life Consumed Analysis on High Temperature, Highly Stressed Structural Parts" by G. W. Weber, Manager, Reliability Applications, Reliability & Design Methods Engineering, Large Jet Engine Department, General Electric Co., Cincinnati, Reference 340.

DEGREE OF VALIDATION

The use of model-sampling methods in high-speed computers has made the processing of distributed quantities in complex calculations relatively simple and straightforward. This permits the full probabilistic simulation of critical design relationships to obtain distributed answers, from which the probability of failure can be deduced. In the example presented, an important economic result was achieved through the use of this method. However, full verification must await demonstration in service usage.

The overall method is by no means completely developed. Further work is needed to improve accuracy and validity in the following areas:

1. Definition of input distributions of load and strength factors.

2. Methods for the identification of fully independent causes of variability, and for the conservative treatment of areas where this cannot be done precisely.

3. More efficient model sampling methods to reduce computer processing costs and to evaluate more accurately the probable sampling error distribution.

However, the state of the art today is such that a critcal design problem can now be approached with confidence that a practical probabilistic solution can be obtained. Conservative approximations of the input distributions can be made where data is lacking, and the cost of further data acquisition can be balanced against the economic value of the anticipated increase in accuracy.

PURPOSE OF TEST: Determination of Probability of Failure of the Structural Casing.

DESCRIPTION OF TEST METHOD

1. Definition of Failure:

The mode of failure is stress rupture, either at a point where a boss is welded into the cylinder or at a high temperature point near the cylinder center, caused by hoop stress due to internal pressure.

2. Methods of Gathering Data

The turbojet engine structural casing is used on a series of engine models with increasing requirements.

Application of the casing to most recent model required an overall time between overhaul (TBO) engine design objective of 1200 hours.

Load - temperature - time pattern varies greatly depending on mission type flown.

Good mission data is available for frequency of type and variation of operating conditions.

3. Accelerating Stresses - pressure and temperature.

SUMMARY OF RESULTS

1. Life Calculations

The structural casing capability was estimated at 600 hours, by "worst case" inputs to the Life Calculation.

The Life Calculation method used permitted no direct calculation of failure rate versus age.

The Life Calculation method used also provided no quantitative means of calculating the effect of load and strength variations on the life and reliability of the part.

The mean engine could survive the mean pattern of load conditions for many thousand hours.

The engine would fail in a very few hours if every strength and load factor were shifted 3 standard deviations in the adverse direction.

Neither of the last two statements is realistic and the answer lies somewhere between them.

2. Former Approaches to Problem Solution

Stress rupture cracks are intergranular and are generated by the combined effect of time, temperature and stress (see Figure 4.5.1).

The Larson-Miller method greatly simplifies the picture in Figure 4.5.1 by reducing the family of curves to a single curve (see Figure 4.5.2).

The Larson-Miller parameter, $P = T(20+\log t)\times 10^{-3}$,

where T = absolute temperature, °R

t = time in hours.

The Cumulative Damage Concept (similar to the Miner-Palmgren Method):

Fraction of life consumed at any constant stress and temperature condition. = $\dfrac{\text{Time at that condition}}{\text{Time-to-stress rupture at that condition.}}$

Increments of life consumed from various stress-temperature states can be summed until unity (or 100% life consumed) is reached, at which point stress rupture can be expected.

The Cumulative Life consumed in Stress Rupture, LCSR, can be obtained from:

$$LCSR = \sum_{i=1}^{n} \frac{t_i}{t_{L_i}} \qquad (4.5.1)$$

where t_i = time in the i^{th} state

t_{L_i} = time to rupture in the i^{th} state

n = number of states or periods of fixed stress and temperature.

This calculation method gives a single, deterministic answer, provides no quantitative estimate of the Probability of failure at a given age and for given distributions of design and operating conditions.

3. The Probabilistic Approach

Conceptually, any deterministic calculation can be converted to a probabilistic calculation if the probability density distribution of every input factor is known (and is independent of all other

factors), and if these distributions can be processed through the calculation to generate a properly distributed answer. Scrutiny of this statement identifies three basic technological problem areas:

a. The distributions of many input factors are essentially unknown.

b. The mathematical task of processing distributions of various shapes through a complex calculation is a formidable one.

c. The initial statement of a problem seldom includes an array of fully independent inputs.

Problem areas a. and c. above are amenable to direct attack; no basic breakthrough in technology is required to permit usefully accurate results. Problem area b. is more difficult. A powerful approximate method, utilizing modern high-speed computers, is now available to the engineer based on model sampling principles, whereby any deterministic calculation can be repeated many times, each time with inputs selected randomly from known input distributions, to generate a probability distribution of answers.

INSTRUCTIONS FOR USE

In the structural casing example, the major simulation task was the simulation of the types of service missions that the engine can be expected to encounter. The types of missions were reduced to seven basic types, including 29 different unique states or "conditions" of temperature and stress. For each of these conditions, the average pressure and temperature was determined for "hot day," "standard day," and "cold day" conditions. Two components of pressure and temperature variations around these central values were estimated; one component which is consistent throughout an engine life, and one component which is consistent throughout a mission, but variable from mission to mission. Discrete percentages of the missions flown on hot, standard, and cold days were established on the basis of temperature data from the U.S. Weather Bureau. The distributions of mission times and relative frequency of the various mission types were established from a sampling of actual field usage data.

The "General Purpose Systems Simulator" computer program, GPSS II, developed by IBM, was used to simulate the missions and calculate the cumulative life consumed values from all of the mission conditions. A master flow chart is shown in Figure 4.5.3.

In Phase A, the dimensional deviations, the material strength deviation and the consistent components of temperature and pressure deviation were selected from their input distributions. In Phase B a specific mission type, a mission time, and the cyclic components of pressure and temperature deviation were selected from their input distributions. In Phase C, the mean pressure and temperature for the first conditions were drawn from a table, the specific deviated values of pressure and

temperature were established, the stress was caluculated, the Larson-Miller parameter value was obtained from a table of parameter versus stress and corrected for the material deviation, and the time to stress rupture was calculated. The fraction of the mission time at this condition was obtained from a table, the time at this condition was calculated, and the ratio of time at condition to the time to stress rupture was calculated and added to the cumulative life consumed in stress rupture. Phase C was repeated (condition loop) until all conditions of this mission were completed. Then Phase B was repeated (mission loop), making a new selection of mission type. This process continued until the cumulative mission time equaled the operating time or life objective. Then Phase A (engine loop) was repeated, initiating a new engine life calculation. The program terminated when the predetermined number of engine lives had been calculated.

Phase D was a manual step wherein the values of life consumed were ordered and plotted as a cumulative distribution for various age levels. At each age level the cumulative probability of failure was estimated from the extrapolation of the life consumed distribution to the 100 percent point (Figure 4.5.5). These intercept values plotted versus age provide a probability of failure versus age characteristic (Figure 4.5.5). Dividing by the age at each point, we get an average failure rate versus age curve (Figure 4.5.6).

The failure rates at the two critical design points were found to be acceptably low at ages well beyond the original TBO objective of 1,200 hr. In fact, based on this calculation these parts should meet the 3,600-hr. ultimate life objective also. Therefore, a costly program to redesign and qualify a new casing was avoided.

LIMITATIONS/RANGE OF APPLICATIONS

The areas of probable greatest error are as follows:

1. Inaccurate input distributions.

2. Imperfection of the life consumed concept.

3. Chance errors in generating the theoretical distribution of results and extrapolating to the 100 percent life consumed value.

This general method is applicable to any mode of failure whose basic failure physics or chemistry is understood, and for which a "failure parameter" can be calculated.

REFERENCES

340

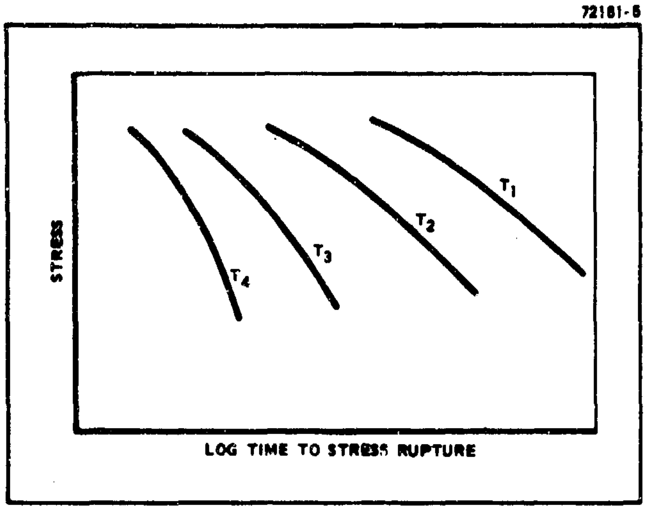

Figure 4.5.1. Stress Rupture vs Time and Temperature

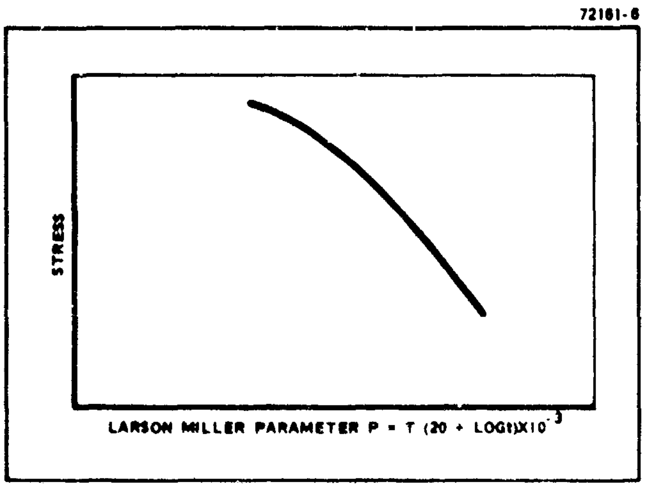

Figure 4.5.2. Stress Rupture vs Larson-Miller Parameter

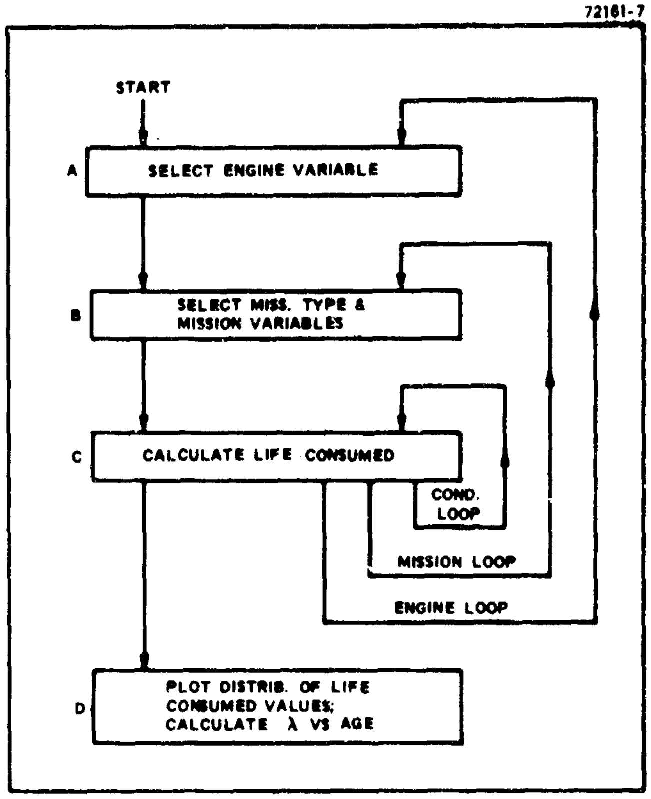

Figure 4.5.3. Calculation Flow Diagram

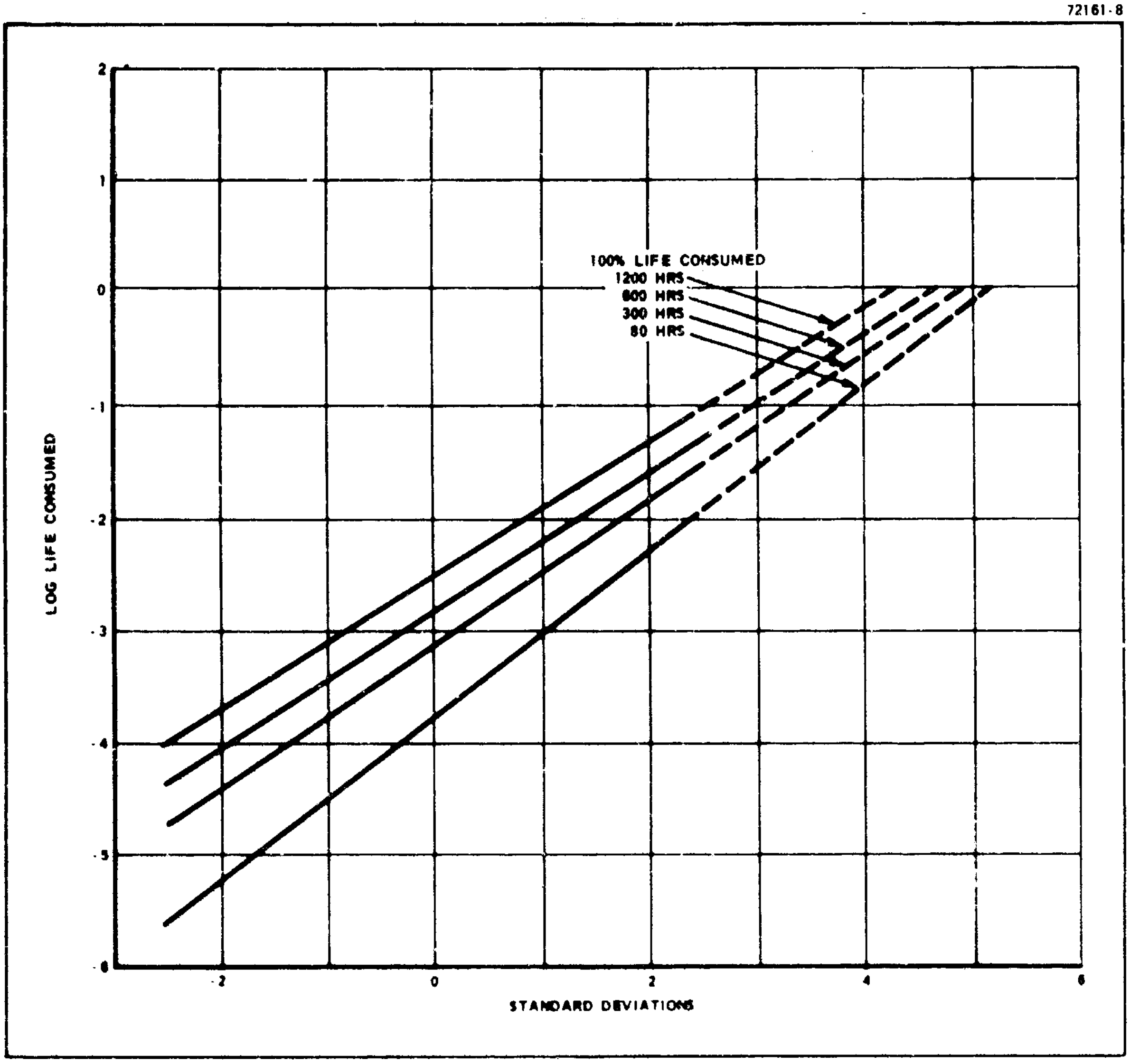

Figure 4.5.4. Distributions of Calculated Life Consumed Values

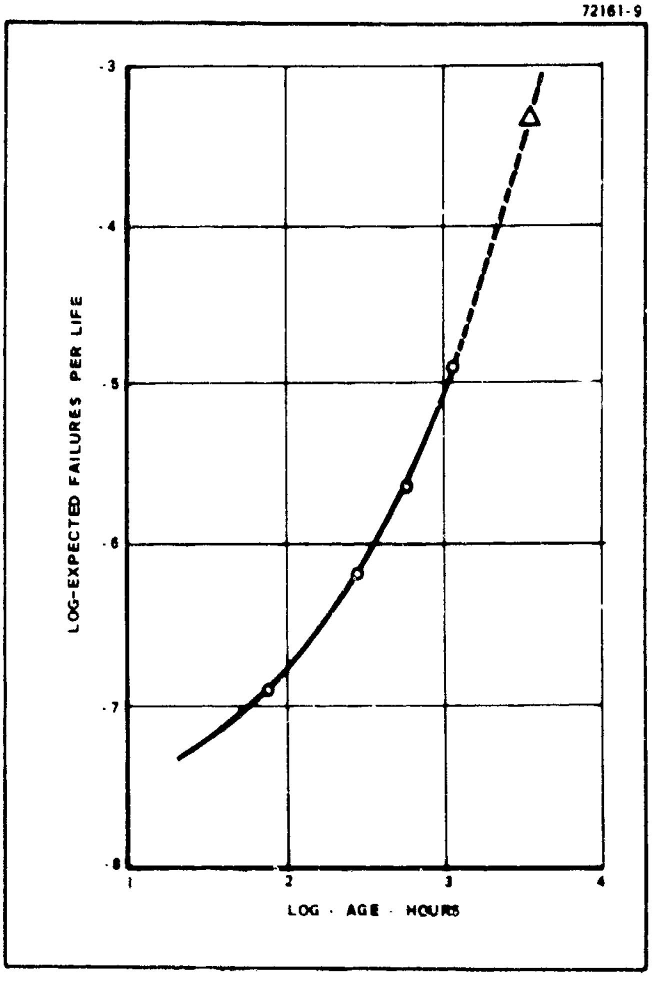

Figure 4.5.5. **Average Failure Rate vs Age Characteristic**

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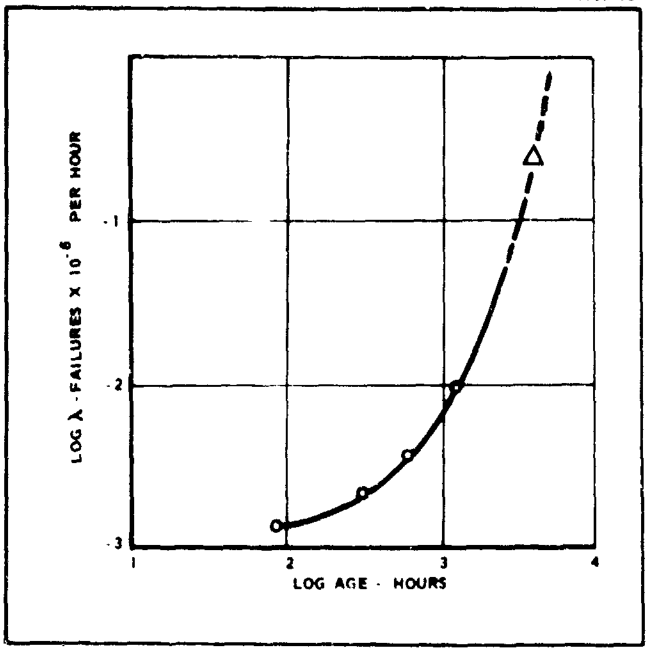

Figure 4.5.6. Probability of Failure vs Age Characteristic

4.6 PART NAME AND DESCRIPTION: Filament Wound Pressure Vessel

ALT Evaluation	
Assumptions Generally Accepted	X
Empirical Proof	
Algorithm	X
Physical Model	

SOURCE: "On the Strength Degradation of Filament Wound Pressure Vessels Subjected to a History of Loading" by Dr. John O. Outwater and Willard J. Seibert, Reference 263.

PURPOSE OF TEST: To predict the ultimate strength of a filament wound pressure vessel that has been subjected to a previous history of loading.

DEGREE OF VALIDATION

The basic theory, mathematical development, and experimentally constructed curves appear to fully complement and mutually support each other, although basic numerical test data from which the curves were constructed is not presented.

DESCRIPTION OF TEST METHOD

1. Definition of Failure:

   The mode of failure is fatigue caused by the hoop stress from the internal pressure within the vessel. The individual filaments deteriorate under load in accordance with Griffith's theory of glass failure.

2. Method of Applying Accelerating Stress:

   The accelerating stress is internal pressure (load) within the pressure vessel. Control of the pressure at different desired levels is attained through use of an Instron testing machine.

SUMMARY OF RESULTS

1. Assumptions:

   The rate of growth of a Griffith crack (microcrack) that controls the strength of a glass fiber (or filament) is proportional to a power of the stress on that fiber.

   Then it can be predicted that:

   a. The ultimate strength of a filament wound pressure vessel decreases linearly with time at a given load.

   b. The time to failure, when the vessel is held at a given load, will decrease logarithmically.

2. Mathematical Models:

a. Consider the walls of a pressure vessel to be made up of bundles of glass filaments each having an initial crack depth C_o.

When the strand is subjected to a stress S for a time t, the crack depth is C.

The rate of growth of the crack is proportional to a power of the stress, S^n (see the basic assumption), or:

$$\frac{dc}{dt} = K_1 S^n \tag{4.6.1}$$

where K_1 is a constant.

Solving Equation 4.6.1:

$$C = K_1 S^n t + C_o \tag{4.6.2}$$

where C_o is the initial crack depth.

By the Griffith theory, the filament will fail when it is kept under a constant stress S when:

$$\frac{K_2}{{}_1S^n t + C_o} \text{ where } K_2 \text{ is a constant.} \tag{4.6.3}$$

Because the initial strength of the glass is much higher than the load at which it ultimately breaks, C_o may be considered small, at the first approximation, compared to $K_1 S^n t$ and the failure load is given by:

$$K_1 S^{n+2} t = K_2 \tag{4.6.4}$$

and:

$$(n+2) \log S + \log t = K_3 \tag{4.6.5}$$

If C_o is not assumed to be small compared to $K_1 S^n t$, then a log $(1-S^2)$ term is present in Equation (4.6.5).

Figure 4.6.1 shows a plot of the load at which a vessel is held, against the time required for the vessel to fail at the load (C_o is considered small). The plot is as predicted by the model, Equation (4.6.5), and the straight line hypothesis is experimentally demonstrated.

b. Determination of the pressure vessel's strength after subjection to a load for a period of time less than required to burst the vessel.

(This is equivalent to showing the vessel's strength deterioration under load. Demonstration is accomplished by rapidly raising the loads to burst on vessels that have been held at a fixed load for different lengths of time.)

If a vessel is held at a load S_1 for a time t, then from Equation (4.6.2), the crack depth of its fibers will be:

$$C = C_o + K_1 S_1^{n} t \qquad (4.6.2)$$

Its breakage load will be given by:

$$S_b^2 = \frac{K_2}{K_1 S_1^{n} t + C_o} \qquad (4.6.6)$$

or:

$$S_b = \left(\frac{K_2^{\frac{1}{2}}}{C_o^{\frac{1}{2}}}\right)\left(\frac{K_1}{C_o} S_1^{n} t + 1\right)^{-\frac{1}{2}} \qquad (4.6.7)$$

In this case the initial crack C_o is not necessarily small compared to the crack depth under load S_1, because the vessel may not have been seriously deteriorated. Therefore:

$$S_b = \left(\frac{K_2^{\frac{1}{2}}}{C_o^{\frac{1}{2}}}\right)\left(1 - \tfrac{1}{2} \bullet \frac{K_1}{C_o} \bullet S_1^{n} t\right) \qquad (4.6.8)$$

and:

$$S_b = S_o - \frac{S_o}{2} \bullet \frac{K_1}{C_o} \bullet S_1^{n} t \qquad (4.6.9)$$

if S_o is the initial glass strength.

Note that in this case the relationship between the breakage stess S_b and time t is linear. Figure 4.6.2 shows this relationship determined experimentally.

INSTRUCTIONS FOR USE

The following description illustrates the methods used by the authors in verifying experimentally the assumptions, the mathematical relationships, and the mathematical models. These methods may be used as instructions for making strength predictions on other glass filament wound pressure vessels.

1. A series of vessels were wound from X-944 glass roving and from E-glass.

2. Load the vessels under internal hydrostatic pressure (a different constant pressure level for each vessel) of high enough level to burst the vessels after a period of time.

3. Calculate the fiber tensile stresses for each vessel at burst.

4. Prepare a log-log plot of the fiber tensile stresses vs. the times to burst, similar to Figure 4.6.1.

5. The Figure 4.6.1 curves show a distinct difference between X-994 and E-glass as far as their bursting strengths are concerned. The curves also show a logarithmic decrement with time in their bursting strengths - giving a straight line curve for each material.

6. The slopes of each curve in Figure 4.6.1 appear the same. By correlating the data of the curves by computer, the value of the exponent n in Equation (4.6.1) can be found. For X-994 and E-glass n was found to be 25.

7. Having established the curves in Figure 4.6.1, burst times can be estimated with the aid of the curves for any condition of load within the range of the curves. Extrapolations may be attempted beyond the upper limit of burst data, but verifying tests are recommended.

8. The next problem is to determine the strength degradation of a vessel as it is exposed to a load of less than its burst strength.

9. Hold each vessel at a load (internal pressure) less than that would burst the vessel, regardless of the length of time it is subjected to the load. Use a different time for each vessel; although the loads may be the same.

10. Rapidly bring the vessels up to their burst pressures, and record the time required and the burst pressures.

11. Calculate the vessel fiber stress for each pressure level, both bursting and non-bursting.

12. Plot curves on linear coordinate paper similar to Figure 4.6.2, which will show the variations in bursting strengths that result from preloading.

13. The authors using Equation (4.6.9) and the curves, found n for X-994 glass again to be 25.

14. The results indicate that on the basis of these tests that the value of the exponent n may be universal for glass of all types - although this must be verified. At least n is the same for X-994 and E-glass, whether it has a pre-load history or not.

Steps 1 through 14 point to a simple method of predicting the strengths of pressure vessels that have been loaded at one pressure for a period of time and then tested to burst at a rapid load or held at another load until they burst.

1. The slope of the burst curves for X-994 and E-glass were found to be the same. Therefore, as in Figure 4.6.3, a universal degradation curve can be plotted at a slope given by -1/n+2 where n = 25 or the slope being -0.037. This line (curve) should be drawn through the rapid load bursting value of stress obtained experimentally.

2. The effect of static fatigue can now be predicted from the curve by obtaining the time to burst under a given load from the intercept of the load with this inclined line.

3. If the degradation under a partial load is desired, then the equivalent of straight lines shown in Figure 4.6.2 should be replotted on the double logarithm curve of Figure 4.6.3. The end point of each degradation curve will be obtained by the time to burst under the same load. The initial point of each curve will be the rapid load breaking point, and points in between will be obtainable by replotting from the straight line curve of Figure 4.6.2.

4. The degradation under a partial load will be obtainable by following the partial curve of the appropriate load for the time prescribed by the actual loading in Figure 4.6.3.

5. The burst pressure after the partial load will be obtainable from the ordinate of Figure 4.6.3.

6. If the vessel is subjected to another pressure, then the ordinate is transferred to another load degradation curve, and that curve is followed for the prescribed time, or followed to burst as in the example shown in Figure 4.6.3.

7. The predicted burst time via this method for a vessel was 750 seconds. The actual burst time was 709 seconds.

8. The data on degradation shown in Figures 4.6.1 and 4.6.2 also follow this prediction system based on the hypothesis in Equation (4.6.1) as illustrated in Figures 4.6.1, 4.6.2, and 4.6.3.

LIMITATIONS/RANGE OF APPLICATION

1. The mathematics and basic theory are limited to glass filament wound pressure vessels, although they appear closely related to the cumulative damage theory for fatigue in metals.

2. The experimental data indicates that the value of the exponent n may be universally 25 for all types of glass, but it must be verified.

3. The method appears applicable to evaluation of filament wound pressure vessels of all types, including rocket motor cases.

REFERENCES: 263.

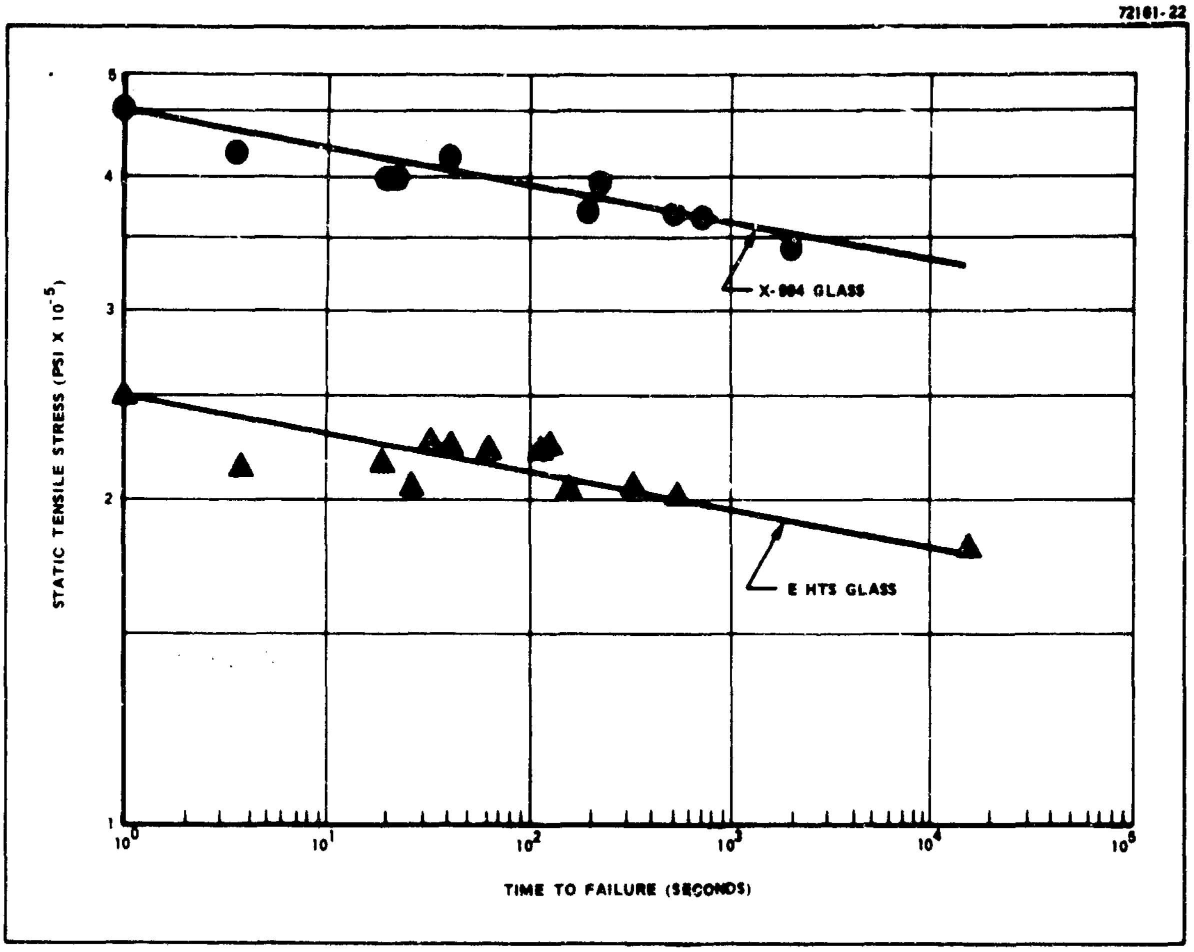

Figure 4.6.1. Time to Failure of Vessels Under Constant Internal Pressure

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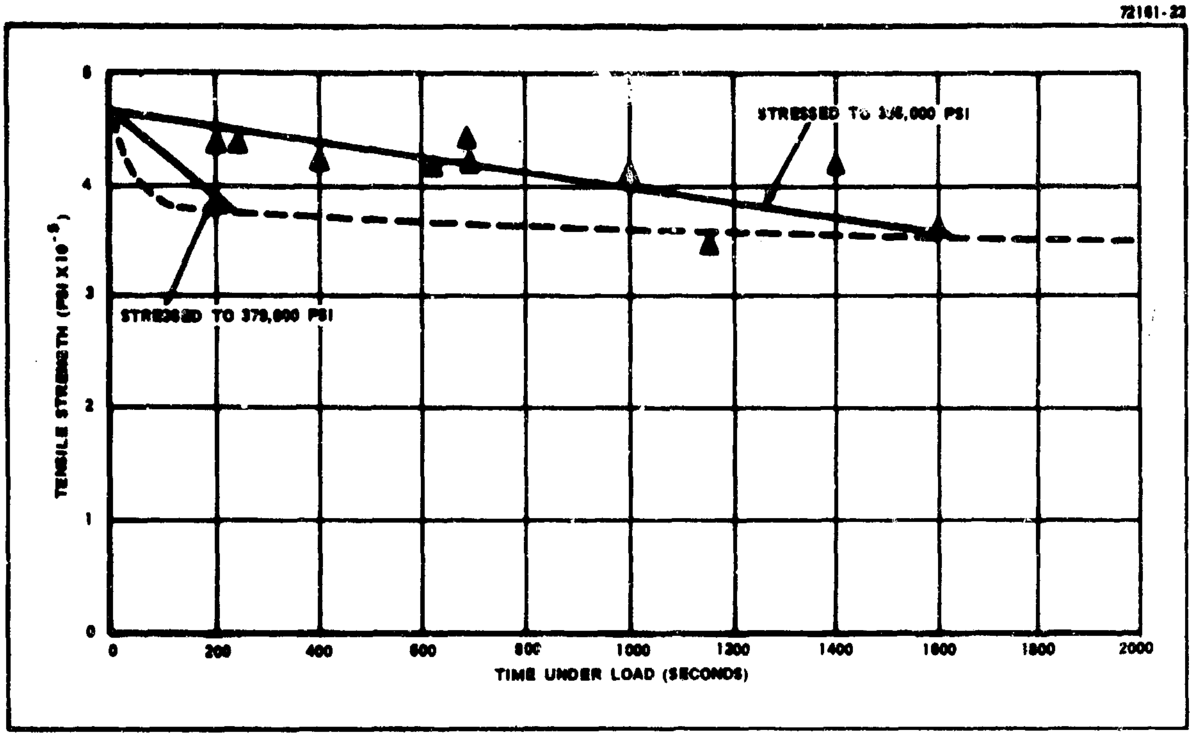

Figure 4.6.2. Rapid Load Bursting Strength of Vessels After They Have Been Subjected to a Partial Load for Varying Periods of Time

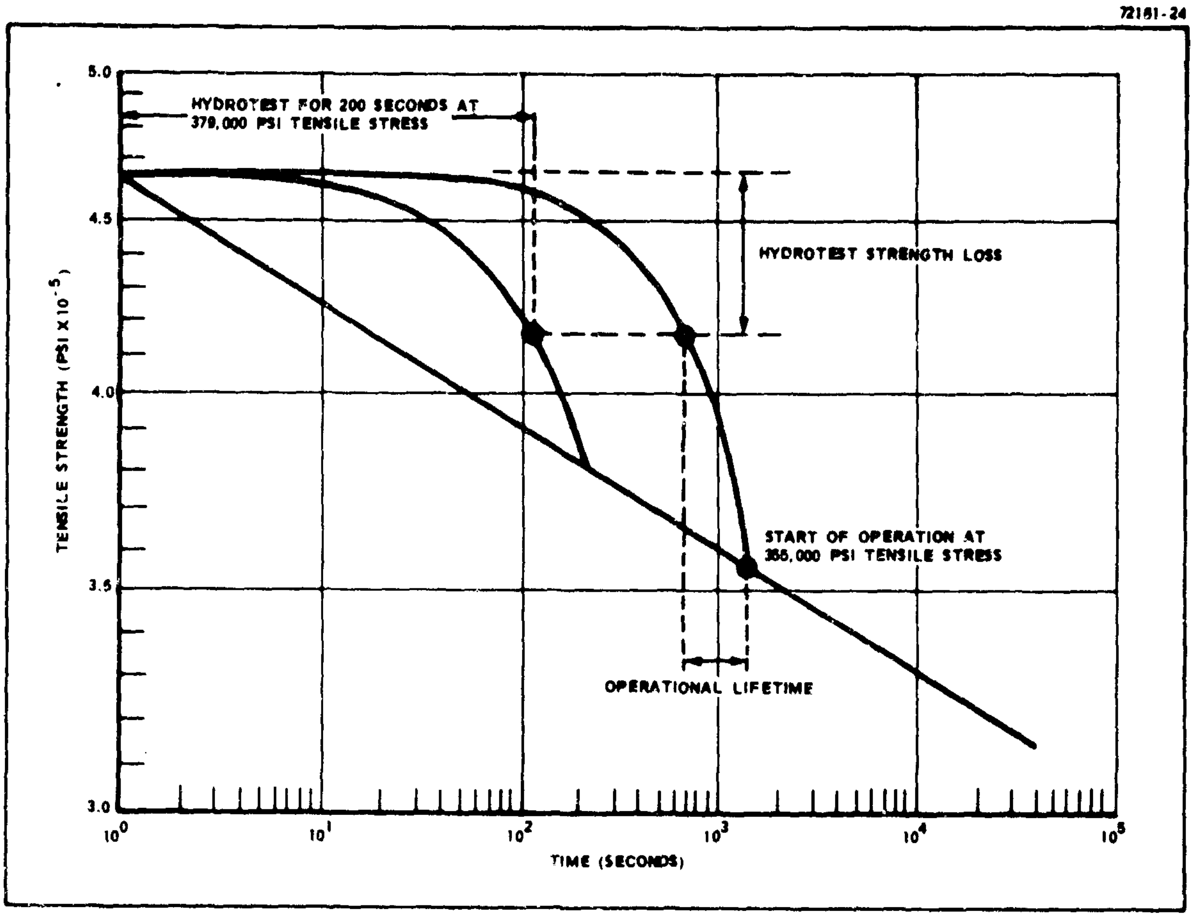

Figure 4.6.3. Chart Combining Figures 4.6.1 and 4.6.2 to Enable Predictions to be Made for the Life of Vessels Under a Constant Working Load Differing From a Short Time Proofing Load

4.7 PART NAME AND DESCRIPTION: Aircraft, Full-Scale Fatigue Test

SOURCE: "Second Seminar on Fatigue and Fatigue Design", AD 611 414 by J. Branger, Reference No. 493.

ALT Evaluation	
Assumptions Generally Accepted	X
Empirical Proof	
Algorithm	
Physical Model	X

PURPOSE OF TEST: To predict fatigue life and factors of safety for full-scale aircraft structures, including wings, fuselage, tail structure, and landing gear.

DEGREE OF VALIDATION

The author presents good experimental evidence of tests on complete aircraft structures which obtained fatigue failures that were found also in service aircraft. Fixes on the test specimens were also appropriate for the aircraft and vice versa. Data and methods are presented for two different types of aircraft. At the time of the report preparation in March 1965, more time was required to verify service aircraft fatigue life and safety factors. The facility is an excellent continuing operation, and full validation and method improvement are the goals toward which the facility is striving.

DESCRIPTION OF TEST METHOD

1. Definition of Failure:

   The mode of failure is fatigue of any part of the aircraft structure, caused by the aircraft operational conditions at any time during its service life.

2. Method of Applying Accelerating Stress:

   A full-scale aircraft structure is subjected to a compressed history of dynamic reversing full-scale stress loads obtained from analyses of types and numbers of missions and the mission profiles expected during its operational life.

3. Test Equipment:

   The Fatigue History Simulator at the Swiss Government's Aircraft Establishment, Flugwerk Emmen, was utilized to conduct the aircraft structure fatigue tests. The Simulator was developed by Mr. J. Branger, Chief Engineer of Flugwerk Emmen. A brief description of the essential elements of the device follows:

   a. Design Requirements:

      1. All loads to be applied to the aircraft structure must be controlled by a single command in such a way that all loads

acting upon the structure at the same time remain always in the same proportion to each other.

2. The test facility must be such that any loading history, especially any sequence of different loads and kinds of different loads, can be applied.

3. The test facility must control itself automatically and provide for a cut-off before fracture of a type not relevant to the test can occur.

4. As far as possible the test facility should be constructed from standard parts.

b. Major Elements:

1. The "Fatigue History", an abstracted or compressed version of the service history of the aircraft, is applied as a fatigue loading to the aircraft structure. The forces are produced hydraulically and transmitted mechanically to the test specimen. The hydraulic forces are controlled electronically.

2. Simple hydraulic jacks produce the forces and may be used in both fatigue and static testing. The jacks can produce tension, compression, or tension and compression forces.

3. The load maintainer, a closed-loop, force servo system, controls the hydraulic pressure to the jacks. The pressure is settable by an adjustable lever fulcrum force balance system within the load maintainer.

4. The main pressure pump supplies hydraulic oil to the system at 2,500 PSI and 25 GPM or more. The high performance of the system is achieved, because the available pump pressure at full flow is $3\frac{1}{2}$ times the system working pressure of 700 PSI.

5. The electronic control system is divided into two systems, one for transmission of orders and one for supervision. The regulating element is a punched tape similar to that on the Telex.

SUMMARY OF RESULTS

1. Mathematical Analysis:

It has been established over the years by many studies and tests that reversing-stress fatigue loading produces differing and non-reversible effects in the material to which the loading is being applied. Therefore, the sequence of the loading is of prime

importance in structural fatigue testing, particularly for airplanes, and has led to the development of "History Loading" as being superior to all program loadings and random loadings.

The below listed terms are applicable to the brief discussion of the scheme of "History Loading" (see Figure 4.7.1).

- Δ the lowest considered step between different magnitudes of loading ($\leq 5\%$ of the ultimate load).
- l_q quasi-cycle, general term for symmetric and asymmetric cycles.
- f_k k^{th} of a random sequence of flight types ($k = 1,\ldots,q$), each simulating an actual flight.
- q period, all flight types of a unique sequence, which can be repeated as often as desired. (How to build up a period is described later.)
- H_q number of quasi-cycles in a period.
- f_q number of flight types, different or uniform, in a period.
- T_q simulated number of service flight hours represented. (Note that the size of loads, their number, and the number of flights are exactly simulated.)

The working scheme is divided into four phases of a number of sequential steps each.

Phase I - Static Tests

a. Determination of all loads.

b. Determination of weight history.

c. Determination of load distribution for the individual load cases, based on wind tunnel data and weight calculation.

d. Summary of loads of the same kind and distribution to single points and the determination of the resulting failure forces for static failure tests, as well as the minimal necessary jacks.

e. Determination of the maximum bending in the static failure test and the resulting required minimal strokes of the jacks.

f. Set-up of the first hydraulic scheme for the jacks loading.

g. Design and construction of the test rig for the particular airplane structure.

h. Preparation of the test specimen for the static measurement tests and for the failure test.

i. Installation of the jacks, whiffletree, and the test specimen, and the connection of the hydraulic load maintainer. Attachment of the defect supervision and manual pressure control devices.

j. Static tests.

Phase II - Program Determination and Test Set-Up

a. Multiflight measurements on a prototype airplane to obtain dynamic behavior.

b. Continuous records of some measurements such as V-g(c.g.)-h-T-G (fuel), stress at certain points, longitudinal and lateral acceleration (for ground loads) in the following kinds of flights:

   1. Test flights by test pilots.

   2. Peace time type of flights by various service pilots under different conditions of weather, season, and route.

   From this data are constructed flight types f_1 to f_q.

c. Statistics (the compiling of all events):

   1. Service flights.

   2. Gust spectra.

   3. Excesses recorded in flight of the permitted limits.

d. Build-up of the program for one period:

   1. Length must be such that the rarest event occurs at least once. From this T_q (the service flight hours) is determined and the f_q (the number of flight types in a period).

   2. From c2, reduced to this T_q, a rough skeleton results for checking the product from b2 and c1.

3. The number of times each flight type exists in a period is now known. All flight types are numbered continuously and mixed according to the Monte Carlo method. The whole sequence of flight types yields the period q, which is transferred to punched tape.

e. A special program from b1 is made and is run only once, and at the beginning, for the new test specimen.

f. Adaptation of the test facility. The facility is adapted to the special requirements of the program q.

g. Preparation of the test specimen - gauges, load attachment points, instruments of supervision.

h. Fitting of the test specimen and connection of the test rig hydraulics and electronics.

Phase III - Dynamic Tests and Analyses

a. Running of the fatigue tests. First simulation of the test flights, and then running the periods q, which are repeated continuously, interrupted only by regular inspections.

b. Determination of virtual advance factor

$$\phi_V = \frac{T_q}{T_V}$$

c. Control of the use of the service aircraft, so that at the beginning their operation remains about equal to each other. Determination of the highest service factor n (effective flying hours per year).

d. Determination of the effective advance factor

$$\phi_W = \frac{T_q}{n \cdot T_V}$$

e. Detected fatigue defects on the test specimen are repaired or reinforced in the same manner as on the operational aircraft.

f. Determination of inspection periods for the operational aircraft.

g. After a non-repairable failure has occurred, the service time Z is affixed, and perhaps a new safety factor ξ against fatigue failures established to get the admissible life time

$$Z_{ADM} = \frac{Z}{\xi}$$

Phase IV - Continued Service Aircraft Surveillance

a. v - g - h records and fatigue meter counts are continued and evaluated on service aircraft as long as they are used. It is possible to fix eventually a new admissible life or a new safety factor, should any change in spectrum or in the operation of the aircraft occur.

2. Factor of Safety Against Fatigue Failure:

a. The factor of safety against fatigue failure is the ratio of the service lifetime Z resulting from the fatigue test to the admissible lifetime Z_{ADM} of the aircraft

$$\left(\xi = \frac{Z}{Z_{ADM}}\right).$$

b. Therefore, a fatigue test is necessary to calculate a factor of safety for the time being. The factor of safety is smaller when the relationship between test and reality is more accurate (ξ_I, imitation factor).

c. The scatter in an entire aircraft (ξ_C, complexity factor) reaches a minimum which is already overshadowed by the evident production tolerances and margins.

d. Therefore, the scatter is smaller according to the steadiness of production and assembly (ξ_P, production factor).

e. The steadier all aircraft concerned are flown and maintained, the smaller the scatter in the fatigue history of the service aircraft (ξ_S, service factor).

f. The loading sequence effect on scatter has not been investigated fully, but it is expected that the lowest scatter is found in "History Loading" (ξ_L, loading sequence factor).

g. To summarize, each of these factors must be evaluated individually, because some time will elapse before there is a sound basis for their calculation. The following table shows an evaluation for the Swiss VENOM aircraft:

Imitation	$\xi_I = 1.1$
Complexity	$\xi_C = 1.1$
Production	$\xi_P = 1.25$
Service	$\xi_S = 1.2$
Loading Sequence	$\xi_L = 1.1$

. Totalling:

$$\xi = 1.1 \times 1.1 \times 1.25 \times 1.2 \times 1.1 = 2.0 \text{ Factor of Safety.}$$

INSTRUCTIONS FOR USE

The instructions-for-use comprise a summary of the author's outline of the full-scale fatigue test of the Swiss DH-112 VENOM aircraft.

1. Problem Statement

   The Swiss Air Force required to use the plane with full armament twice the flying time of that permitted by the fatigue restrictions. The manufacturer's guaranteed safe life was 500 flying hours based on fatigue tests.

2. Terms

   To solve the problem the following terms were set up:

   a. The load conditions of the aircraft in service had to be ascertained.

   b. These load conditions had to be represented in full-scale test without restriction.

   c. The test structure had to be in strict accordance with flying aircraft, including all mods.

   d. Incipient cracks and their propagation had to be observed in the full-scale test - and periodic intervals for service aircraft established on that basis.

   e. All service aircraft had to be put into service in such a way that all were fatigued in the same manner to form a single population.

3. Support and Load Application

   The test aircraft is loaded in such a way that it floats and all forces are in equilibrium. It is connected to the ground through the wing/fuselage connecting bolts; thereby, leaving the landing gear free, so that it can be loaded separately by jacks.

4. Statistics of Fatigue Loads

   Excellent fatigue data is available because of a program started in 1954 to instrument service aircraft with fatigue meters and accelerometers. Vertical and longitudinal accelerations were measured on service aircraft for these tests including landing and taxying.

A statistics skeleton of the fatigue meters was prepared from the above data (see Figure 4.7.2), converted to 1,000 hours.

5. The Test Program

A "History Loading", a fatigue loading in the form of up and down loads in accordance with the actual conditions and sequence for the VENOM, was prepared in the following manner:

a. The Period

The load limit of 6.5 g is exceeded only on special command and then only by manual control; therefore 6.5 g is the maximum acceleration to be included in the automatic test program. With the estimated occurrence of $N = 5$ for $n = 6.5$ g in 1,000 hours, the spectrum can at most be divisible by 5, and the minimum period is 200 flying hours. The 200 hours represent 350 flights with an average of 34.3 minutes per flight from Air Force data.

b. The Weight

The take-off and landing weights were summarized, in accordance with the fuel carried, into five groups: 5/5, 4/5, 2/5, and 1/5 full tankage. The statistics yield 8 weight series:

189 take-offs with 5/5 of full tankage; of these
172 landings with 1/5 tankage
* 10 landings with 2/5 tankage
* 7 landings with 3/5 tankage

150 take-offs with 4/5 of full tankage; of these
140 landings with 1/5 tankage
* 7 landings with 2/5 tankage
* 3 landings with 3/5 tankage

11 take-offs with 3/5 of full tankage; of these
7 landings with 1/5 tankage
* 4 landings with 2/5 tankage

with * indicating 31 flights at gusty weather conditions.

c. The Flight Type Phases

The flight type phases analyzed, sorted and simulated from records are:

1. Take-off (ground cases) including refueling, taxying and take-off up to unstick.

2. Flying (air cases): Normal and gusty, and take-off weights.

3. Landing (ground cases): Touch down and taxying up, landing weights, speeds, and braking.

4. Taxying back (ground cases): Short and long, and different loads.

d. The "History Loading"

The above flight phases were composed of 350 different flights. The sequence of the flight types was chosen by the Monte Carlo method and put on a single Telex tape, which contains more than 90,000 orders on a length of 235 meters (770 feet) and represents a period of 200 flying hours. By running the tape 5 times through the machine, 1,000 flying hours are represented.

e. The Test Run

The working speed of the Fatigue History Simulator is controlled by the rig itself: The switch to the next order occurs only after the present order has been completed. The VENOM test tape ran through the machine once in 120 hours (5 days) for 200 hours flying time. Inspection took about 15 hours.

The total run of the spectrum (1,000 hours) without an additional one or two weeks added for normal Air Force servicing was 28 days.

At 472 hours the wing was reinforced at Rib #2 in the same manner as for service craft, about 20% of which were developing cracks due to undercarriage load.

At 472 and 672 hours fatigue cracks were observed in the rear wing portion and repaired. Similar cracks were found in service craft and repaired in the same manner.

At 1,472 hours a starboard fuselage/wing attachment fitting was replaced by one from an aircraft which had developed a fatigue crack. At the same time a redesigned fitting was installed on the port side. The cracked fitting crack length increased from 0.25" to 0.40" in 600 hours and stopped. The redesigned fitting did not crack. Because it was found not necessary to replace the fitting on service aircraft, the savings were equivalent to 25% of the full-scale tests.

The wing attachment one-piece bolt pitted very easily and was replaced at 400 hours with one with a split sleeve. The defect was also found on service craft. Replacement required costly wing removal. The redesign was installed on all service craft, thereby saving all future wing removals.

The test was stopped at 5,000 hours and the specimen exposed to an overload of 9 g's - 90% of ultimate load. It did not fail. Dismantled wings showed some easily repairable failures.

f. Inspection Methods

1. External visual every 200 hours.

2. Servicing of the aircraft including removal of wings, access doors, and wing attachment fittings at 1500 hours.

3. Dy-check and magnetic crack detecting methods on fittings.

4. Television cameras at critical wing structure points.

5. Wing deformation is measured to indicate if an important failure has occurred on the structure at 200 hours at a static 3 g's load.

6. Noise within the wing is recorded by a very sensitive tape recorder and compared with original recordings.

7. The machine-applied orders (all) are counted, and the load of the hydraulic jacks are periodically recorded.

8. Strain gage measurements are made of the tension-compression loads of the main wing spar.

9. A diagrammatic illuminated picture is set up before the aircraft to show the modes of loading in color coded lights at the proper relative positions.

g. Test Results and Future Aspects

1. As implied in the ALT, the tests were continuing in March 1965 at the time of the author's presentation of his study. As of that date no serious fatigue failures had been produced.

2. The VENOM aircraft could at that time be declared safe for 1,200 hours, which is 200 hours more than the 1,000 hours required for extended use by the Swiss Air Force, and more than twice the original guaranteed safe life of 500 hours.

3. Further tests will be run on a new pair of wings to determine the time of main spar failure. These tests will be verified on two more pair of wings.

4. The most important results of the future tests will be the establishment of the scatter to define the coefficient of safety against fatigue failure.

LIMITATIONS/RANGE OF APPLICATION

1. Certain structural parts of aircraft, depending on the type, are unobservable by any means during a full-scale test. These parts must be determined and separate subassembly fatigue tests run.

2. Better and automatic inspection methods are needed to reduce the time required for visual inspections and insure complete inspection of all required surfaces and hidden parts to indicate the start of fatigue cracks. An electronic device, such as the Sigma-Test apparatus of Dr. Forster (Germany), could be adapted to help greatly.

   The author points out that the outputs of thousands of these devices, which utilize AC current to generate an alternating magnetic field in a coil, which in turn, generates eddy currents in the specimen, could be fed into a computer and stored and compared separately. A change of the specimen surface conductivity (with the start of a fatigue crack) exceeding a specified limit would cause the computer to give an indication that the failure was taking place at a certain location on the specimen.

3. Although specifically developed for aircraft structures, for which the need of such a method is most urgent and necessary, the same system is adaptable to other structures, such as trucks, autos, road machinery, small boats and submarines, and other beam structures of numerous varieties, including small bridges or portions of bridges, buildings, and cranes.

REFERENCES: 493.

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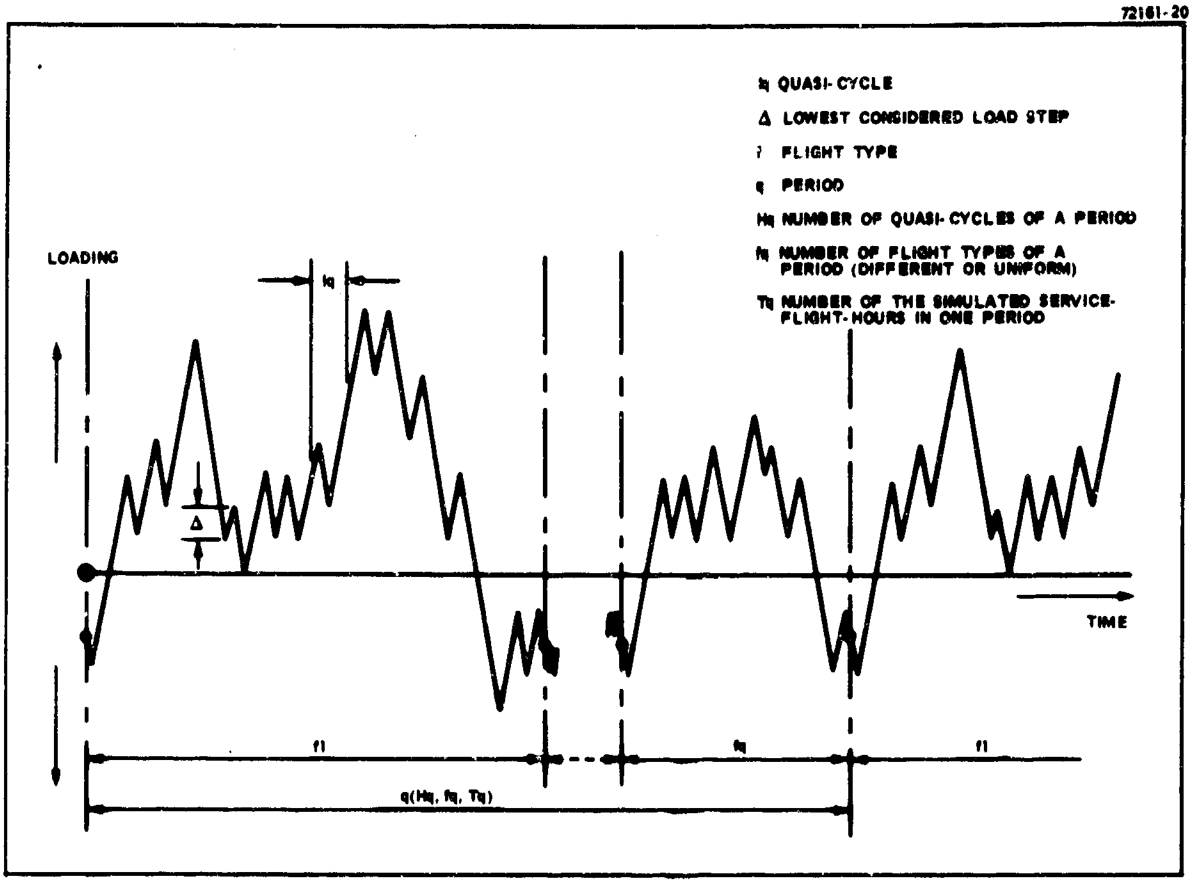


Figure 4.7.1. Scheme of History Loading

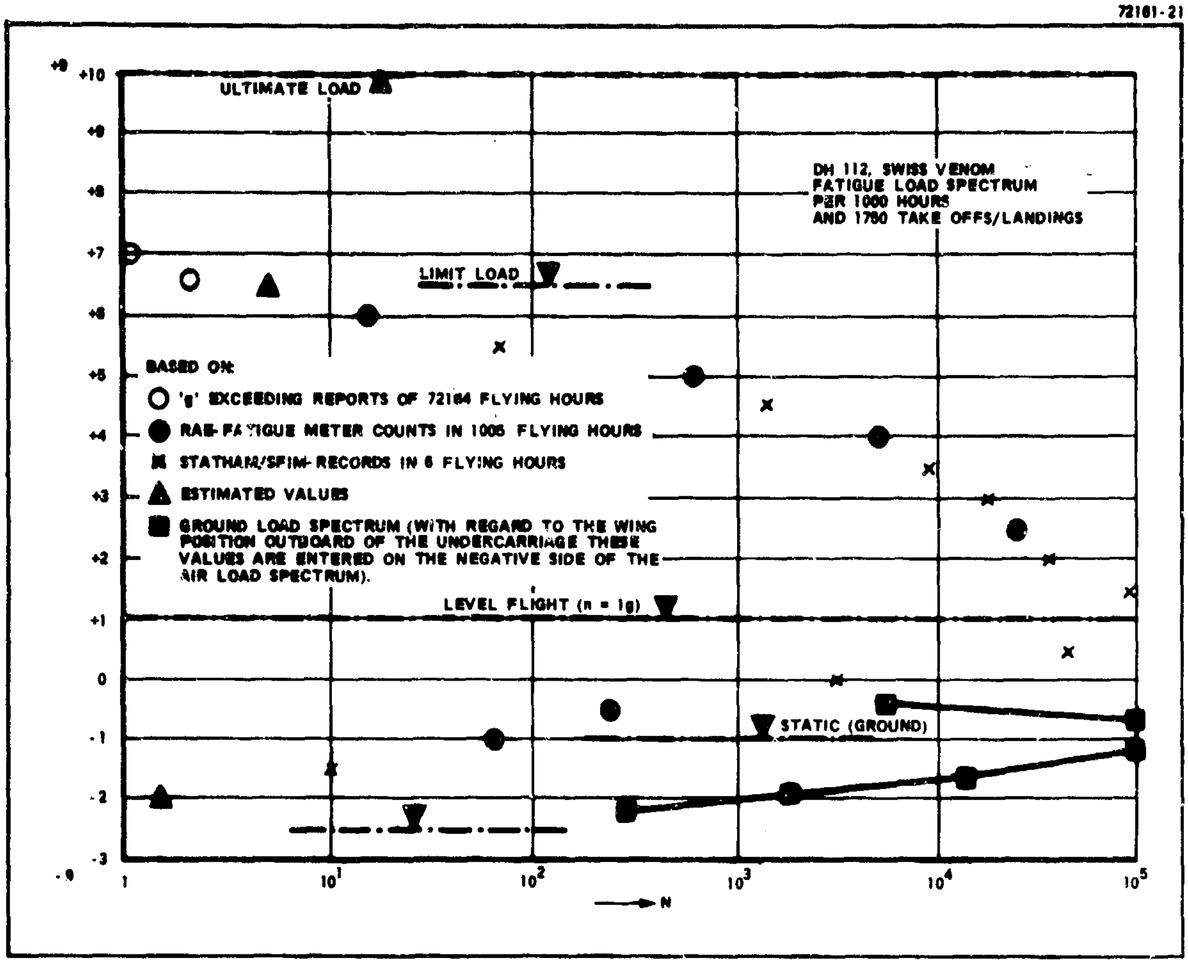


Figure 4.7.2. Fatigue Load Spectrum DH-112

4.8 PART NAME AND DESCRIPTION: Mechanical Seals (O-Rings)

AN6227B-11; OD-3/4", ID-9/16", cross sectional width 3/32"; material: Buna-N Rubber; applied as a flanged cover recessed gasket seal against hydraulic pressure.

ALT Evaluation	
Assumptions Generally Accepted	X
Empirical Proof	X
Algorithm	X
Physical Model	

SOURCE: "Final Report For Accelerated Reliability Test Methods For Mechanical and Electromechanical Parts", RADC-TR-65-46, January 1965, Reference No. 298.

PURPOSE OF TEST: To develop an ALT method based on time transformations of the distribution functions of parts tested to failure at overstress conditions to yield quantitative estimates of reliability characteristics at rated stress levels.

DEGREE OF VALIDATION

Validation of this ALT is not complete, because the schedule of the study program upon which it is based did not permit life tests of the O-rings at normal conditions of temperature. Therefore, of the three proposed models developed in the study, which included ALT's for other types of parts, it was not possible to determine the model applicable to the O-rings tested.

DESCRIPTION OF TEST METHOD

1. Definition of Failure

In the application as a flanged cover recessed gasket seal against hydraulic oil static pressure, the definition of O-ring failure is that there will be no leakage for three test pressure cycles of one minute duration each, during which the pressure on the test fixture is raised to 1500 PSIG and held for one minute and returned to 0 PSIG. The mode of failure is radial fracture or cracks and/or circumferential cracks in the O-ring.

2. Method of Applying Accelerating Stresses

a. Stress Selection

Very little information relating mechanical life to environmental stresses was available. An investigation of potentially usable accelerating stresses indicated the following general properties:

Tear resistance	fair
Abrasion resistance	good
Aging (Ultraviolet)	fair
Oxidation (Ozone)	fair
Resistance to compression set	good
Oil and gasoline resistance	excellent
Acid resistance	good
Cold resistance (freezing)	good
Heat resistance	good
Permeability to gases	medium
Electrical resistivity	low
Resistance to cutting	good
Resistance to water swelling	excellent

An examination of the above properties resulted in the elimination of those that would be difficult to repeat with confidence. Also eliminated were stresses that would require removal of the rings from their test block, because handling would be a difficult-to-control condition. Since a combined environment test was an objective of the project, known non-compatible tests were ruled out.

This resulted in an initial accelerating stress selection of:

Aging (Ultraviolet)

Oxidation (Ozone)

Heat Resistance

While conducting tests to determine stress levels to be utilized, it was found that total breakdown of the ozone occurred at temperatures above 200°F. This resulted in elimination of ozone as an accelerating stress. References indicated that Buna N rubber compounds are relatively insensitive to damage at temperatures below 250°F, therefore samples were tested at 300°F, and it was found that failure occurred in less than 88 hours.

It was felt that measurement error resulting from the combinations of heating and cooling cycles involved in the testing procedure would be too great with this short length of life span. Subsequent tests and information led to the specification of 275°F as the upper limit, 200°F as the lower limit and an intermediate condition of 250°F. All available information indicated that a lower temperature than 200°F would not produce failures within the contractual period. The level of the ultraviolet exposure was determined from a study of manufacturers data on the light sources.

A summary of the overall stress selection is:

Applied Stress	Normal	Intermediate	Maximum
Temperature	200°F	250°F	275°F
Ultraviolet Exposure	None	0.1 watt/ft^2	0.2 watt/ft^2

b. Stress Application

A 2^3 factorial statistical experiment with 10 replications was selected for the study program on O-rings. This was to permit evaluation of the main effects of temperature and ultraviolet as well as their interactions when applied as combined stresses. Table 4.8.1 describes the different combinations of test conditions included in the test program.

As previously stated, it was impossible to operate the parts at temperatures lower than 200°F and still be able to observe a sufficient number of failures during the contractual period to perform an analysis. However, ranges of temperature and ultraviolet exposure were included as test conditions which would yield a general idea as to the manner in which life varied as stress levels were varied.

TABLE 4.8.1. FACTORIAL EXPERIMENT FOR O-RINGS

Temperature (°F)	Ultraviolet Exposure		
	None	.1 Watt/Ft.^{2}	.2 Watt/Ft.^{2}
200	(1)	(2)	(3)
250	(4)	(5)	(6)
275	(7)	(8)	(9)

Note: The numbers in parentheses are designations used throughout the study to identify each of the nine test runs. For example Test Run #5 consists of 10 O-rings which were tested at 250°F and .1 Watt/Ft.^{2}. Each test run contained 10 O-rings.

c. Test Equipment and Physical Test Method

Figure 4.8.1 illustrates the method of environmental stress application. The outline represents the temperature controlled oven, which was set for the required temperature stress level. The chamber was set up to simultaneously supply the three levels of ultraviolet exposure by using an opaque shield for the "no ultraviolet" level, setting the bulb-to-O-ring distance for 0.2 watt/ft^2, and providing a $\sqrt{2}$ distance-from-bulb relationship for the 0.1 watt/ft^2 and 0.2 watt/ft^2 levels. The bulb-to-O-ring distance was periodically reduced to compensate for the light output decrease associated with bulb aging.

The O-rings were installed in individual test blocks and were not removed until failure was evident. This minimized the possibility of damage by mishandling or from installation and removal. The test blocks were mounted on the pressure test fixture illustrated by Figure 4.8.2. A hydraulic pressure of 1500 psi was applied for a period of 1 minute and then released. The cycle was repeated 3 times for each test cycle. The O-ring and test block were removed as a unit and washed with gasoline to remove the hydraulic oil. After drying, O-ring and test blocks were placed in the appropriate environment chamber. Periodically the test specimens were removed from the environment chamber, allowed to cool, pressure tested, cleaned and returned to the environment chamber. This was repeated until failure occurred.

The following items of test equipment were used in the testing procedure:

Item	Function
Tenny Oven 520-011528	Temperature environment
Dispatch Oven Serial #52517	Temperature environment
Ultraviolet Lamp G8T5	Ultraviolet environment
Crosby Model AIH 1500 psi gage	Indicate test pressure
Blackhawk Model P151 Hydraulic Pump	Hydraulic Test Pressure Source

SUMMARY OF RESULTS

The study to find an accelerated reliability test method for O-rings was performed by testing 10 parts at each of 9 different combinations of two types of stresses: temperature and ultraviolet exposure. Each stress was applied at 3 levels of severity. The mean of the tests on individual parts at each of the combined stresses is shown in Table 4.8.2.

1. Analysis of Variance

Before an analysis of variance could be performed on the O-ring life test data, a logarithmic transformation had to be made on the data. The reason for this was that the variance of the longer life test runs was much higher than those tested at accelerated conditions. The analysis of variance indicated that both temperature and ultraviolet and their interactions affected the life of O-rings. The details of the effects are shown in Table 4.8.3.

The F Ratio indicates that both temperature, ultraviolet exposure, and their interactions are highly significant. This is evaluated by comparing the values in the column in Table 4.8.3 marked $F_{.05}$ to the values in the F Ratio column. A larger number in the F Ratio column denotes significance. The Components of Variance analysis substantiates the values of the F Ratio but points out that temperature exerts an extremely powerful effect on the life of these parts (about 98.4% of the observed variance). Regardless of the overwhelming temperature effect, the effects of ultraviolet and their interactions are large enough to be felt. The right hand column in Table 4.8.3 gives a complete list of the % contribution to total variance of all the effects.

TABLE 4.8.2. O-RINGS: HOURS TO FAILURE

Temperature	No Ultraviolet	0.1 Watts Per Ft.^{2}	0.2 Watts Per Ft.^{2}
200°F	$\bar{X}$ = 1751.4	$\bar{X}$ = 1395.4	$\bar{X}$ = 1297.0
250°F	$\bar{X}$ = 327.9	$\bar{X}$ = 340.9	$\bar{X}$ = 347.3
275°F	$\bar{X}$ = 148.7	$\bar{X}$ = 136.4	$\bar{X}$ = 134.8

TABLE 4.8.3. ANALYSIS OF VARIANCE - O-RINGS

Source of Variance	Sum of Squares	Degrees of Freedom	Mean Squares	F Ratio	$F_{.05}$	% Contribution To Variance
Between temperatures	15.90668	2	7.95334	4791.2*	4.00	98.4
Between Ultra-violet Levels	.08211	2	.04106	24.7*	4.00	.5
Interaction Between Temperature and Ultraviolet	.03199	4	.01600	9.6*	2.53	.5
Residual	.13473	81	.00166			.6
Total	16.15551	89				100.0 %

*Denotes significance at the $F_{.05}$ level.

2. Failure Analysis of O-Rings

Each O-ring, after failing the pressure test, was inspected under a microscope to determine the nature of the failure mode. There are only two failure modes that appeared predominantly. They were:

1. Radial fracture or deep radial cracking.

2. Circumferential cracks usually on the outside diameter of the ring.

The circumferential cracks that appear occasionally throughout all but Test Run 4 were located randomly along the outer surfaces. They usually, but not always, were connected to deeper radial cracks. The circumferential cracks periodically crossed the mold line on the outer diameter of the O-ring. The severity and frequency of both the radial and the circumferential cracks did not seem to form a pattern that increased in severity with stress. The lone exception to this was at 275°F where a larger proportional amount of circumferential cracks occurred.

3. Analysis Related to Life Distribution Functions

The failure data for O-rings was plotted on the Weibull probability paper. The lines of best fit were calculated on the computer by the method of least squares. The Weibull shape (β) and scale (α)

parameters were calculated and are summarized in Table 4.8.5. Alpha (α_k) is the coded value corresponding to the scale parameter that can be obtained from the Weibull plot of each test run. Alpha (α_o) is the uncoded value of the scale parameter.

TABLE 4.8.5. SUMMARY OF WEIBULL PARAMETERS FROM O-RING TESTS

Temperature	Ultraviolet Exposure		
(^oF)	No UV	.1 Watt/Ft^2	.2 Watt/Ft^2
200	β = 17.75 α_3 = 19,930 α_o = 35,436 x 10^{53}	β = 10.71 α_3 = 48,424 α_o = 65,324 x 10^{29}	β = 11.01 α_3 = 22,646 α_o = 24,277 x 10^{30}
250	β = 8.46 α_2 = 102,740 α_o = 85,069 x 10^{17}	β = 8.12 α_2 = 25,330 α_o = 43,618 x 10^{16}	β = 22.46 α_2 = 2314 x 10^9 α_o = 19,248 x 10^{41}
275	β = 14.68 α_2 = 403.43 α_o = 92,426 x 10^{27}	β = 7.94 α_2 = 15.487 α_o = 119,111 x 10^{12}	β = 5.17 α_2 = 4.95 α_o = 106,326 x 10^6

4. Selection of Accelerated Test Method (O-Rings)

It would appear that any of the stresses induced during the O-ring tests would result in an acceleration of the failure of the parts. Temperature exerts by far the most significant effect on the life of O-rings, but ultraviolet has demonstrated its usefulness also as an accelerating stress. The true validation of whether or not the results found in this study are translatable to the life of O-rings at some other range of temperatures cannot be answered. The basic relationships shown here doubtless vary for parts produced from different materials. However, the main objective of the test has been accomplished -- namely demonstrating how the life of a given type of O-ring varies when subjected to a given application, under various combinations of ultraviolet and temperature stresses. It has also been demonstrated that a combined environments test involving both temperature and ultraviolet results in an added reduction in life that would not be in effect if the temperature and ultraviolet were applied separately or sequentially.

INSTRUCTIONS FOR USE

1. Step-By-Step Physical Test Procedure

The specific steps in the O-ring test procedure are as follows:

Visually inspect O-ring for manufacturing imperfections and carefully install in test block with identification dots on the exposed surface.

Mount O-ring and test block on the pressure test fixture and raise hydraulic pressure to 1500 psi for approximately 1 minute. If O-ring passes initial pressure test, remove block and ring from test fixture, wash in pan of gasoline and carefully blot with a clean cloth until dry.

Record environmental exposure information on data sheet. Place O-ring and test block in oven and expose to the proper level of ultraviolet light.

None - Place in shield area

*0.1 Watt/Ft^2 - set block in position 14" from bulb

*0.2 Watt/Ft^2 - set bulb at 9.9" above O-ring

Reduce the vertical height of the bulb by 0.1" each 7 day period to compensate for bulb aging. Replace the bulb at 10 week intervals with a new bulb that has been "burned in" for 100 hours.

O-rings exposed to the *275°F environment shall be pressure tested at 24 hour maximum intervals until failure. Pressure testing shall consist of following the initial test procedure except it shall be pressurized to 1500 psi for three cycles for each test.

O-rings exposed to the *250°F environment shall be pressure tested 48 hour maximum intervals in the same manner as the 275°F O-rings.

O-rings exposed to the *200°F environment shall be pressure tested at maximum 4 day intervals in the same manner as the 275°F O-rings.

When failure is evident, remove the O-ring from the test block, wash in gasoline, and place in an envelope having the identification information on the exterior.

* These were the values used in the Reference 298 study; other values of temperature and ultraviolet exposure may be chosen by the user.

2. Mathematical Procedure

a. Perform the accelerated tests for 3 levels of temperature, all greater than normal (room), and 3 levels of ultraviolet, including 0 and 2 higher, superposed on temperature.

b. Record hours to failure for O-rings in a matrix in accordance with Table 4.8.2; that is, with levels of ultraviolet for columns and levels of temperature for rows.

c. Find the arithmetic mean ($\bar{X}$) of each cell.

d. Perform logarithmic transformation of data to determine effects of temperature and ultraviolet on life of O-rings by determining by the components of variance analysis as shown in Table 4.8.3. If the F ratio numbers are larger than $F_{.05}$ in all three cases, then the effects of temperature and ultraviolet are significant for the levels chosen to test.

e. Perform the t tests to observe how the interactions are grouped, in order to select those combinations of stresses causing differences to occur in the mean lives, as shown below in steps 1, 2, and 3.

   1. Tabulate the logarithmic means of each of the 9 O-ring test runs as shown in Table 4.8.4.

   2. Calculate $\bar{X}_1-\bar{X}_2$, the interval between the logarithmic means required for significant differences to exist, based on $t_{.05}$ with 81 degrees of freedom and the residual mean square from the analysis of variance in Table 4.8.3.

   3. Form the logarithmic means into 6 groups as described by the temperature and ultraviolet exposure levels, and analyze the results with respect to the $\bar{X}_1-\bar{X}_2$ interval found in 2 and the effects of the temperature and ultraviolet exposure.

f. Analyze the failure modes (in this case there were only two: radial and circumferential cracks) to insure that exposure to some levels of temperature and/or ultraviolet do not create a third failure mode. Data for a third mode of failure must be discarded and the ALT terminated just below that level of environment (accelerated stress).

g. Perform the analysis related to life cycle distribution functions by performing the following steps:

   1. Plot the O-ring failure data for each cell of the matrix of Table 4.8.2 on Weibull probability paper. For accuracy, the lines of best fit are calculated on the computer by the method of least squares.

   2. The Weibull shape (β) and scale (α) parameters are calculated and summarized in accordance with Table 4.8.5.

3. Mathematical Models

Reference 298 proposes three mathematical models (Models 2, 4, 5) as possible descriptors of the relationship between accelerated

and normal conditions. A mathematical model, which yields repeatable results, is a prime requirement for a useful accelerated test, because the math models yield an algorithm for translating Weibull parameters observed at accelerated conditions to the Weibull parameters (to be) observed at normal conditions. Figure 4.8.3 presents the models.

Unfortunately, in the O-ring program, available contract time did not permit tests at normal conditions (room temperature); therefore, which of the models is applicable to the O-rings tested is not presently determinable.

The models would be used to predict α_A^* and β_A^* to see if the predictions fell within the confidence limits calculated from already observed results at the normal conditions.

The user of the ALT would be equipped with an estimate of an old accelerated and old normal set of Weibull shape and scale parameters as defined in the table. He would obtain these estimates from an initial normal and accelerated test run made on his own test equipment and the parts of interest. Any time an estimate was required of the life expectancy of the parts when operated at manufacturers' rated conditions, the user would simply test a sample of current production parts at accelerated stress conditions and obtain the estimates $\tilde{\alpha}_A^*$, and $\tilde{\beta}_A^*$. The estimates could be combined with the previous estimates $\tilde{\alpha}_N$, $\tilde{\beta}_N$, $\tilde{\alpha}_A$, and $\tilde{\beta}_A$ in the algorithm to solve for the new estimates $\tilde{\alpha}_N^*$ and $\tilde{\beta}_N^*$. These could be used to construct the cumulative failure distribution and used for estimates at any desired probability of survival or mission time.

The results obtained would represent an estimate of the life expectancy of the parts from the current production run or lot. The estimate could be obtained by performing a relatively short test, thereby saving test time and test expenses.

It is assumed that the $\tilde{\alpha}_N$ and $\tilde{\beta}_N$ generated in the referenced study represent estimates for parts produced only by those respective manufacturers. The mathematical model should be translatable to usefulness for similar parts produced by other manufacturers and to parts in the same generic family. However, proof of the model's range of applicability are not contained in the study.

FIGURE 4.8.3. Algorithm for Calculating Life Expectancy at Manufacturer's Rated Stress Conditions $\tilde{\alpha}_N^*$, $\tilde{\beta}_N^*$

Model 2	$\tilde{\alpha}_N^* = \tilde{\alpha}_A^* \Big/ \left(\dfrac{\tilde{\alpha}_A}{\tilde{\alpha}_N}\right)^{\tilde{\beta}_A^* / \tilde{\beta}_A}$	$\tilde{\beta}_N^* = \dfrac{\tilde{\beta}_N}{\tilde{\beta}_A} \tilde{\beta}_A^*$
Model 4	$\tilde{\alpha}_N^* = \dfrac{\tilde{\alpha}_A^* \tilde{\beta}_N^*}{\tilde{\beta}_A^*} \Big/ \left(\dfrac{\tilde{\alpha}_A \tilde{\beta}_N}{\tilde{\alpha}_N \tilde{\beta}_A}\right)^{\frac{\beta_A^* - 1}{\beta_A - 1}}$	$\tilde{\beta}_N^* = \left(\dfrac{\tilde{\beta}_N - 1}{\tilde{\beta}_A - 1}\right)(\tilde{\beta}_A^* - 1) + 1$
Model 5	$\tilde{\alpha}_N^* = \left(\dfrac{\tilde{\alpha}_N \tilde{\beta}_A}{\tilde{\alpha}_A \tilde{\beta}_N}\right) \dfrac{\tilde{\alpha}_A^* \tilde{\beta}_N^*}{\tilde{\beta}_A^*}$	$\tilde{\beta}_N^* = \tilde{\beta}_A^* + (\tilde{\beta}_N - \tilde{\beta}_A)$

where:

$\tilde{\alpha}_N^*$ = estimate of Weibull scale parameter for parts if they had been tested at manufacturer's rated operating and environmental stresses

$\tilde{\beta}_N^*$ = estimate of Weibull shape parameter for parts if they had been tested at manufacturer's rated operating and environmental stresses

$\tilde{\alpha}_A^*$ = estimate of Weibull scale parameter obtained from a current test run at accelerated stresses

$\tilde{\beta}_A^*$ = estimate of Weibull shape parameter obtained from a current test run at accelerated stresses

$\tilde{\alpha}_N$ = estimate of Weibull scale parameters from a previous test run of parts operated at manufacturer's rated conditions

$\tilde{\beta}_N$ = estimate of Weibull shape parameter from a previous test run of switches operated at manufacturer's rated conditions

$\tilde{\alpha}_A$ = estimate of Weibull scale parameter from a previous test run of switches operated at accelerated stress conditions

$\tilde{\beta}_A$ = estimate of Weibull shape parameter from a previous test run of switches operated at accelerated stress conditions.

LIMITATIONS/RANGE OF APPLICATION

1. The ALT physical test methods and analyses techniques are generally applicable to a wide range of similar parts and different generic families.

2. The ALT is unique in that it includes two different types of accelerating stress, one superposed on the other - in this case ultraviolet light exposure added to temperature. Analysis techniques are presented for determining the effects of each type of stress, singly and combined, on the O-ring life. These techniques, or related ones, have a wide range of important applications in today's products, which are subjected to many different types of "multi"-environments in aerospace systems and others.

3. The ALT is specifically limited in the particular instance of an O-ring of Buna-N rubber material, in that a material change to one of silicone or other could have profound effects on the failure modes and causes of failure (environment), with resulting possible changes of test equipment types, kinds of accelerating stresses, test procedures, and perhaps even the analysis techniques and mathematical models.

REFERENCES: 298.

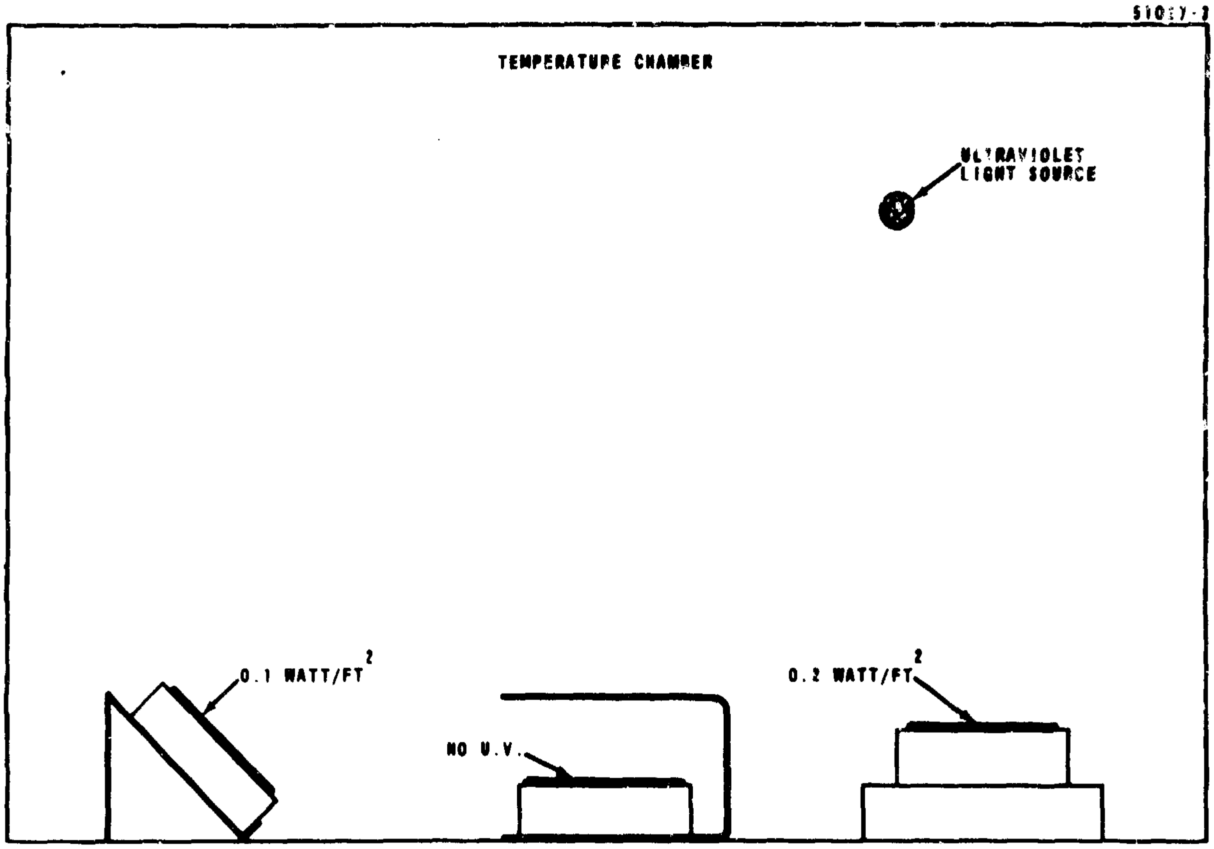


Figure 4.8.1. O-Ring Test Method

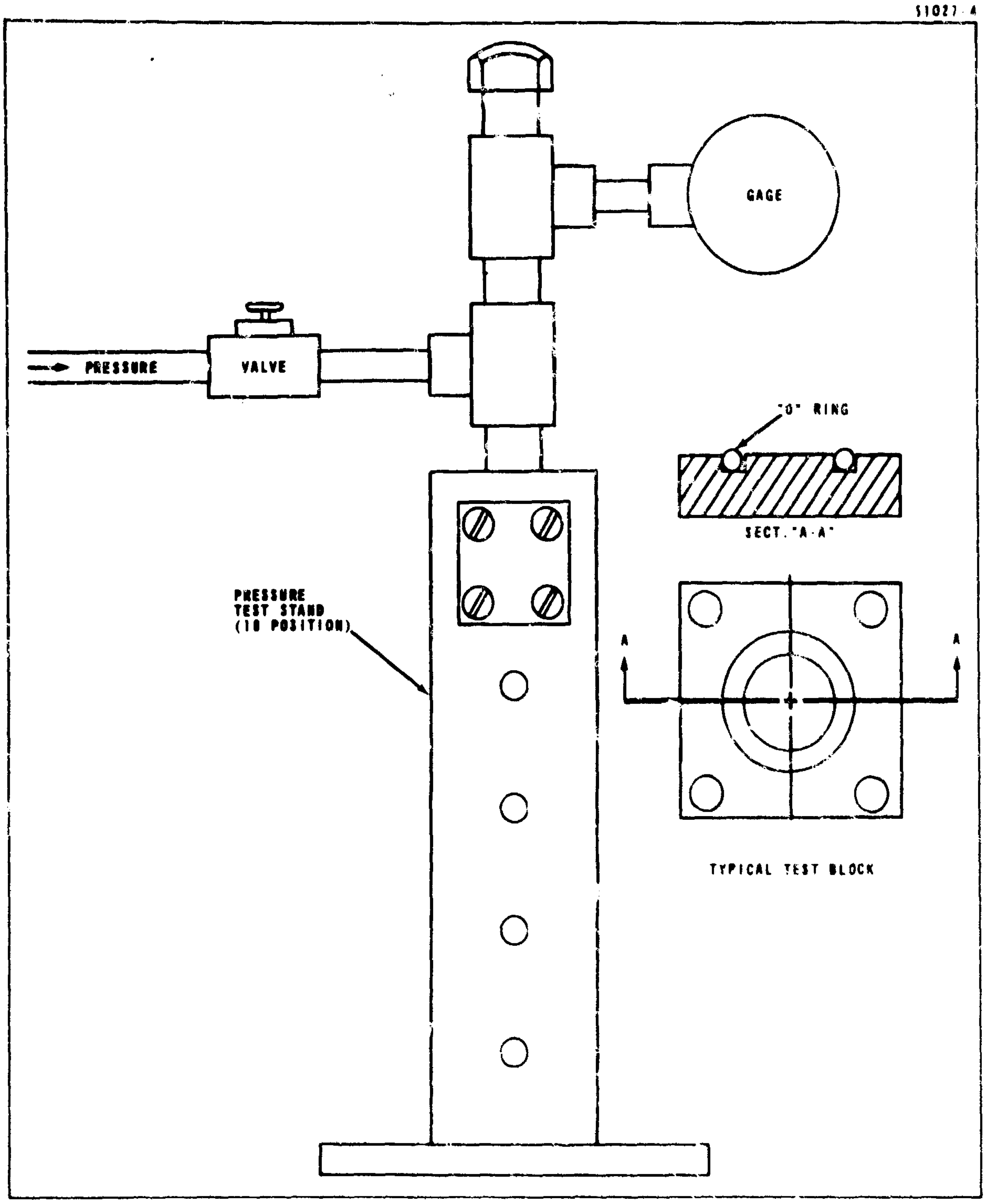


Figure 4.8.2. O-Ring Test Fixture

5.0 MORE POWERFUL STATISTICAL METHODS

The uniform format described below was used because it supplies all the information necessary to utilize a more powerful statistical method (MPSM).

The information items shown below were developed for each MPSM described in this section. The length of the discussion varies from MPSM to MPSM but the treatment is uniform.

The first two data items identify the method. The titles are the authors or were prepared for this handbook. The source is the original paper and is the basic reference for the MPSM.

Items 3, 4, 5 and 6 are of great importance in selecting an MPSM. The purpose of the MPSM indicates the parameters that can be treated and the decision processes that can be implemented. The limitations, Item 4, supply information on situations where the MPSM cannot be utilized. This information is important if misapplication of the method is to be avoided. Item 5 identifies the criteria used to measure efficiency and states the reasons the method is more powerful. Before selecting a MPSM, it is necessary to be sure the method produces the correct information and minimizes the expenditure of the most valuable resources. Finally, before an MPSM can be selected it is necessary to know the limitations and what knowledge is required in its use. If the required knowledge is available and the limitations are tolerable, the MPSM can be used. If the knowledge is unavailable, another method may be more suitable or the knowledge will have to be obtained at additional cost.

The instructions for using the method are given in Item 7. These data items are step-by-step instructions for utilizing the MPSM. Tables are not presented in this handbook, but references to needed tables are given in Item 7.

The advantages indicate what quantitative improvement or savings are possible using the method. Item 9 comments on any experience accumulated with the method.

- 1.0 Title of Method (Author's Title).
- 2.0 Source.
- 3.0 Purpose.
- 4.0 Limitations of Method.
- 5.0 Criteria Used in Measuring Efficiency of Method.
- 6.0 Knowledge Required to Use Method.
- 7.0 Instructions for Use.
- 8.0 Advantages of Method.
- 9.0 Reported Experience with Method.

5.1 INTRODUCTION TO BAYESIAN METHODS

Topics 5.1.1 through 5.1.13 present Bayes methods of solving various reliability estimation problems. Several points should be mentioned about these sections.

First and perhaps foremost, the Bayes methods will quite probably take a very important place in future Reliability estimation and demonstration procedures. However, because of the addition of a prior distribution, the Bayes methods entail perhaps greater problems of mathematical tractability than the classical methods. Virtually all of the papers of this section represent evidence of the common device of making simplifying (and unverified) assumptions for the sake of obtaining mathematical tractability (e.g., the assumption that the prior distribution is discrete with all its mass at two points). As the Bayes methods become more accepted, more sophisticated research is to be expected. In fact, a rather good defense to the possible criticism of oversimplified assumptions is that one must first interest the user in the approach in an understandable and easy to use way.

The second point appears to be inescapable: One must have knowledge of the prior distribution of "reliability." This is perhaps the biggest stumbling block for using Bayes methods. It is well-known in statistics that "ignorance," in general, costs money, i.e., test time, test costs, etc. If one desires to benefit from Bayes methods, one must learn about the prior distribution on reliability.

In general, Bayes sampling plans by attributes (i.e., lot sampling for fraction defective) have not been written up because Reliability is usually concerned with life times. However, References 413 and 430 furnish Bayes attributes sampling plans.

The reader will find wide disparities in the notation used among the papers written up. However, this well reflects the philosophy used in this handbook.

1. Decide if the work should be written up at all. Naturally, some judgement is involved here but the ones not written up are available in the bibliography.

2. The works written up were merely reported on. No improvements were attempted.

Thus, each write up is intended to be a self-contained methodological presentation of the particular paper. To change notations so that a common notation was possible would have made reference back to the original paper difficult for the user.

Finally, it turned out that very often the authors' reasons why the Bayes method is "more powerful" were given somewhat obscurely. However, in general, the Bayes methods are superior because if a prior distribution exists at all (and in Reliability the prior does exist) then the Bayes methods are correct models.

Some discussion is in order of the "meaning" of Bayes confidence intervals as compared to the meaning of classical confidence intervals. The confusion arises because, when put on paper, the two intervals (Bayes and Classical) and their associated confidence levels look very much alike. Some individuals think they are identical (the confidence statements that is, not necessarily the method of arriving at them) others know they are different but are vague as to how they differ. For sake of illustration, let μ be a parameter of a population for which a confidence interval is desired. Before giving an illustration of the difference let the result be known:

> The classical method gives "confidence" that μ lies in the interval constructed but does not, definitely not, give the probability that μ lies in the interval.
>
> The Bayes method gives the probability that μ actually lies in the interval.

To elaborate, consider the classical method of obtaining confidence intervals. It should be noted that the classical method involves inductive inference. Suppose given a set of data D (e.g., $\bar{x}$, $\hat{\theta}$ or some other sample statistic). The classical method computes conditional probabilities $P(D|\mu)$ as μ varies. Now consider the set of all μ which satisfy $P(D|\mu) \geq \alpha$. Let this set be bounded by (μ_*, μ^*). This is inductive inference since one agrees to include in the interval all μ for which the Data D is not too unusual. One then says there is $1-\alpha$ confidence that $(\mu_* \leq \mu \leq \mu^*)$. If it stopped there, everything would be fine. But usually the above procedure is formally written down as:

$$P(\mu_* \leq \mu \leq \mu^*) = 1 - \alpha \tag{1}$$

However, all that the above procedure entitles one to say is that in the long run roughly $(1-\alpha)\%$ of the intervals constructed in this way will contain μ. This statement is not identical with (1). Thus, the word confidence instead of probability. Although α may have been a probability to start, $(1-\alpha)$ after forming the set (μ_*, μ^*) is definitely not. Note that $(1-\alpha)$ should not be used as a probability in any further calculations.

Now the Bayes procedure seeks to find (μ_*, μ^*) such that

$$P(\mu_* \leq \mu \leq \mu^*|D) = 1 - \alpha \tag{2}$$

That is, if the Bayes procedure is workable it gives an actual probability statement on μ. Now clearly to compute (2)

$$P(\mu_* \leq \mu \leq \mu^* | D) = 1 - \alpha = \int_{\mu_*}^{\mu^*} f(\mu|D)\, d\mu$$

one needs the prior distribution $g(\mu)$ because

$$f(\mu|D) = \frac{h(\mu,D)}{P(D)} = \frac{k(D|\mu)g(\mu)}{P(D)}$$

and this is just a form of Bayes theorem. Actually, Bayes may be getting a little more than his due since $f(\mu|D)$ is only a conditional probability. The point is the "Bayes" procedure gives an exact probability statement as against the classical "confidence" statement. The price? - knowledge of the prior distribution on μ.

One might well ask then why are the Bayes intervals called confidence intervals instead of what they really are - probability intervals. The answer is (probably) because if not called confidence intervals people might become confused as to their purpose.

Thus, when a classical demonstration test has been successfully passed (with β and min. acceptable MTBF θ_1) one may say I am 1 - β confident $\theta \geq \theta_1$. One may not say

$$P(\theta \geq \theta_1) = 1 - \beta$$

Now, when a Bayes test is finished successfully (with β^*, θ_1) one is entitled to say

$$P(\theta \geq \theta_1) = 1 - \beta^*$$

Why are claims made that Bayes procedures often reduce test time? Suppose $\beta = \beta^*$; if the prior distribution $g(\theta)$ indicates as relatively unlikely a value of $\theta < \theta_1$ then less testing is required. However, it is only fair to say if $g(\theta)$ indicates $(\theta \geq \theta_1)$ as unlikely the testing may be more than the classical tests.

The criteria/purpose matrix given below enables a user to select a method depending on what one wants to do (purpose) and what criteria of goodness" one selects. If criteria or purposes are missing from the matrix it is because nothing was found in these areas.

Criteria	Point Estimate of Reliability	Interval Estimate of Reliability	Point Estimate of MTBF (failure rate)	Interval Estimate of MTBF (failure rate)	Choosing Prior Distribution	Selecting Reliability Demonstration Test Plan
	PURPOSE OF METHOD					
Test Time, T	5.1.9	5.1.9		5.1.10 5.1.11 5.1.12		
Cost of Test						
Expectation of Loss Function			5.1.4	5.1.4 5.1.13		
Type or Width of Interval	5.1.3 5.1.7 5.1.8	5.1.7 5.1.8	5.1.5	5.1.5		
Mean Square Error	5.1.6				5.1.2	

5.1.1

1.0 Title of Method (Author's Title): Bayesian Statistics for the Reliability Engineer.

2.0 Source: Proceedings of National Symposium on Reliability and Quality, IEEE, 1966, pp. 315-320, A. W. Drake.

3.0 Purpose: The purpose of this paper is a tutorial note on Bayesian methods. It is very readable and is recommended for anyone "thinking" about using the Bayes methods presented in this handbook.

4.0 Limitations of Method: Not applicable.

5.0 Criteria Used in Measuring Efficiency of Method: Not applicable.

6.0 Knowledge Required to Use Method: Not applicable.

7.0 Instructions for Use: Not applicable.

8.0 Advantages of Method: Not applicable.

9.0 Reported Experience with Method: Not applicable.

5.1.2

1.0 Title of Method: Selection of Prior Distribution on Parameters of Time to Failure Distribution by Maximum Entropy Principle.

2.0 Source

2.1 M. Tribus, The Use of Maximum Entropy in the Estimation of Reliability,Recent Developments in Information and Decision Processes, MacMillan Co., N.Y.

2.2 R. A. Babillis and A. M. Smith, Application of Bayesian Statistics in Reliability Measurement, Annals of Reliability and Maintainability, #52

3.0 Purpose

This method establishes prior distributions for parameters that use all available information and still have minimum prejudice. i.e., The distribution does not employ any unstated assumptions or assumptions unsubstantiated by knowledge. The method establishes the distribution of the parameter, say α, when only expectations of functions of α are available. e.g., When the range $E(r(\alpha))$ ($r(\alpha) = 1; \alpha_L > \alpha < \alpha_u$; α_L is the lower limit and α_u is the upper limit), mean $E(\alpha)$, second moment $E(\alpha^2)$ expected log of α $E(\ln \alpha)$, third moment $E(\alpha^3)$, etc. are available but the distribution type is unknown.

4.0 Limitations of Method

4.1 The truncated normal which results from a common situation is very difficult to work with if the time to failure distribution of the part is exponential or gamma.

4.2 The gamma distribution result (although mathematically desirable) is hard to realize because $E(\ln \alpha)$ is usually unknown.

5.0 Criteria Used in Measuring Efficiency of Method

This method leads to the lowest probability of making a drastic error because of the prejudice of the statistician. The priors obtained can be used to lower testing requirements over non-Bayesian tests.

6.0 Required Knowledge to Use Method

To use this method it is necessary to know

(1) What expectations are available.

(2) The values of available expectations.

This method should not be used when the distribution type is known unless the knowledge is converted into moments of the distribution.

7.0 Instructions for Use

Step 1 - Establish which expectations are known.

Step 2 - Use the following distributions when appropriate expectations are available.

Expectation Available	Distribution Density
$E(r(\alpha))$, (α_L and α_u)	Uniform $f(x) = \dfrac{dx}{\alpha_u - \alpha_L}$
$E(r(\alpha))$, $E(\alpha)$,	Exponential $f(x) = \dfrac{1}{k} \dfrac{dx}{E(\alpha)} \exp\left(\dfrac{-x}{E(\alpha)}\right) \qquad \alpha_L \leq x \leq \alpha_u$ when $k = \left(\exp\left(\dfrac{-\alpha_L}{E(\alpha)}\right) - \exp\left(\dfrac{-\alpha_u}{E(\alpha)}\right)\right)$
$E(r(\alpha))$, $E(\alpha)$, $E(\alpha - E(\alpha))^2$	Truncated Normal $f(x) = \exp(-a_o - a_1 x - a_2 x^2)dx$ where a_o, a_1, and a_2 are functions of $E(\alpha)$ and $E(\alpha - E(\alpha))^2$ that are described in the source pp. 116 to 118. In the source $E(\alpha)$ is L and $E(\alpha - E(\alpha))^2$ is called σ^2. Assume $\alpha_L = 0$, $\alpha_u = \infty$.
$E(r(\alpha))$, $E(\alpha)$, $E(\ln\alpha)$	Gamma $f(x) = \dfrac{a^b}{\Gamma(b)} x^{b-1} \exp(-ax)\ dx$ where $E(\alpha) = b/a$ $E(\ln \alpha) = \dfrac{\Gamma'(b)}{\Gamma(b)}$ $\Gamma'(b)/\Gamma(b)$ is the Digamma function which is tabled in reference (a) and (b)

The result of Step 2 is prior distribution having maximum entropy of information.

Reference (a) Handbook of Math Functions, etc., National Bureau of Standards Applied Math Series 55(1964).

(b) H. T. Davis, Tables of Higher Math Functions, 2 Vols., Principia Press, Bloomington, Ind., 1933, 1935.

8.0 Advantage of Method

8.1 It allows one to determine a prior distribution from knowledge of expectations only

8.2 The prior distribution does not depend upon any unstated assumptions about the type of distribution.

8.3 An interesting case (not developed in the references) is one where the mean is known and $\alpha_L \neq 0$. In this case we have

$$f(x)dx = \frac{\exp\left(\frac{-x}{E(\alpha)}\right) \quad dx}{\left[1 - \exp\left(\frac{-\alpha_L}{E(\alpha)}\right)\right]E(\alpha)}$$

which is a displaced exponential. This covers cases where a minimum value of α can be established.

9.0 Reported Experience with Method: Examples are given in the source.

5.1.3

1.0 Title: Point Estimation from Available Information.

2.0 Source: M. Tribus, Use of Maximum Entropy in Estimation of Reliability, Recent Developments in Information and Decision Processes, Macmillan Co., New York, pp. 102-140.

3.0 Purpose:

3.1 Estimation of density of time to failure, τ, and any and all statistics of the distribution of τ.

3.2 Examples

Density of τ; $Pr(t < \tau \leq t + dt) = f(t)dt$

Reliable life (R_{90}); $t \ni Pr\ (\tau \geq t) = .90$

Mean life (θ); $\int_0^\infty tf(t)dt$

3.3 The method can be used in all point estimation problems for an item.

4.0 Limitations of Method

The limitation on this method is that the most common case where information is available on range, θ, and σ^2 leads to the truncated normal distribution that is difficult to manipulate. Also, the truncated normal is not regarded as typical time to failure distribution for electronic parts.

The results when the range and θ information is the only data available are mathematically tractable, however, these cases where information is so limited are very rare and are of little practical significance.

Finally, the theory is not developed to the point where results are available for higher moments; e.g., If the coefficient of skewness is known there are no results available for finding the density.

5.0 Criteria Used in Measuring Efficiency of Method

This MPSM is more powerful because it supplies a logical method for applying prior information even if this information is not a result of direct experimentation. For example, an estimate of mean life is sufficient to supply all the statistics of the time to failure distribution.

6.0 Knowledge Required to Use Method

To use this method, it is necessary to know

6.1 Expectations of functions of τ (e.g., the mean value $E(\tau)$, the second moment $E(\tau^2)$, the range of τ $E(r(\tau))$ where $r(\tau) = 1$; $(T_L \leq \tau \leq T_u)$ and zero elsewhere $\left[T_L\right.$ may equal 0 and T_u may be $\left.\text{infinite}\right]$.

6.2 Which expectations are available for making the estimate. (All the available estimates should be used.)

7.0 Instructions for Use

7.1 Find the Density of τ.

The density function complete with parameters to be used in the following likely cases are

7.1.1 Expectations available = Range (T_u finite)

Density function $Pr(t \leq \tau \leq t + dt) = f(t)dt = \dfrac{dt}{(T_u - T_L)}$

This is a uniform density.

7.1.2 Expectations available.

Range $\left(T_L = 0 \;\; T_u = \infty\right)$ Mean $= \theta$

Density function $f(t)dt = \dfrac{1}{\theta} \exp(-t/\theta)dt$

This is an exponential density.

7.1.3 Expectations available

Range $\left(T_L = 0 \;\; T_u = \infty\right)$ Mean (θ) Variance σ^2

Density $f(t)dt = e^{-(a_0 + a_1 t + a_2 t^2)}dt$

a_0, a_1, and a_2 are found from generalized graphs in the reference.

Example (from the reference) $\theta = 1$; $\sigma^2 = .01$

$f(t)dt = (50/\pi)^{1/2}\, e^{-(50.41 - 100t + 50t^2)}dt$

This is a truncated Normal density.

7.2 Compute Required Point Estimates, e.g.,

7.2.1 Mean life, θ, point estimate

$$\theta = \int_0^\infty tf(t)dt$$

In most cases θ was already available and formed the basis of the analysis.

7.2.2 Probability of survival for time x (Reliability)

$$P_S(x) = 1 - \int_0^x f(t)dt$$

7.2.2.1 Range only; $P_S(x) = 1 - \dfrac{x - T_L}{T_u - T_L}$

7.2.2.2 Information on range and θ; $P_S(x) = e^{-(x/\theta)}$

7.2.2.3 Information on range, θ, and σ^2;

$$P_S(x) = \frac{1-\text{erf}\left[\left(a_1 x/2\alpha\right) + \alpha\right]}{1-\text{erf}(\alpha)}$$

where $\alpha = a_1/2\sqrt{a_2}$ and erf (x) is the standard normal of (x) a_1 and a_2 are determined from the graphs in the reference

7.2.3 Reliable Life R_{90}

$$\int_0^{R_{90}} f(t)dt = .10$$

7.2.3.1 Information available on range

$$R_{90} = T_L + .10(T_u - T_L)$$

7.2.3.2 Range and θ

$$R_{90} = \theta \ln(.90)$$

7.2.3.3 Range, θ, and σ^2

R_{90} cannot be written in closed form but can be obtained from basic equation and tables of the standardized normal. These tables are available in most texts on Math Statistics and many handbooks.

8.0 Advantages of Method

The advantages of this method are that it allows point estimation of statistics used in reliability from limited available information which have minimum prejudice of the tatistician to bias the result.

9.0 Reported Experience with Method: None.

5.1.4

1.0 Title: Bayesian MTBF Warranty Values.

2.0 Source: R. J. Schulhof and D. L. Lindstrom, Application of Bayesian Statistics in Reliability, IEEE, National Symposium on Reliability and Quality Control, 1966.

3.0 Purpose: To establish the warranty MTBF of an item; that is, the value of MTBF the manufacturer will guarantee, for three loss functions.

4.0 Limitations of Method

To use this method requires a lot of a priori information.

i) shape of time to failure distribution,
ii) parameters of the Gamma distribution, and
iii) knowledge of consequences of high and low estimates of d.

5.0 Criteria Used in Measuring Efficiency of Method

The method produces a warranty MTBF that minimizes expected loss based on a priori and test data.

6.0 Knowledge Required to Use this Method

6.1 The loss function which is appropriate.

6.2 The parameters of the a priori Gamma distribution of failure rate.

6.3 The time to failure of the item is exponential.

7.0 Instructions for Use

7.1 Choose one of these three loss functions:

(a) $L(\theta,d) = \dfrac{(\theta-d)^2}{\theta^2}$

(b) $L(\theta,d) = \dfrac{(\theta-d)^2}{\theta}$

(c) $L(\theta,d) = (\theta-d)^2$

where θ is the true MTBF and d is the warranty MTBF (similar to θ_1 of Mil-Std. 781A). Loss Function (a) is to be used when the loss due to a high estimate for d is very great, (b) is used when losses due to high and low estimates for d are about equal and (c) is used when a low estimate for d causes a very great loss.

7.2 Choose d_1 from the following

<u>Loss Function</u>

$$d = \frac{T+\alpha}{(x+1)+\beta} \text{ for use in 7.1.(a)}$$

$$d = \frac{T+\alpha}{x+\beta} \text{ for use in 7.1.(b)}$$

$$d = \frac{T+\alpha}{(x-1)+\beta} \text{ for use in 7.1.(c)}$$

where α and β are the parameters of the prior distribution

$$f(\lambda) = \frac{\alpha}{\Gamma(\beta)} (\alpha\lambda)^{\beta-1} \exp(-\alpha\lambda)$$

λ is the failure rate

T is the total test time on the item and x is the number of failures observed.

8.0 Advantages of Method

The method makes use of prior data and finds without extensive computation warranty MTBF's (d's) that minimize expected loss functions. The loss functions should satisfy a wide variety of situations.

9.0 Reported Experience with Method: None

5.1.5

1.0 Title of Method (Author's Title): Bayesian Approach to Life Testing and Reliability Estimation, S. K. B. Bhattacharya.

2.0 Source: Journal of the American Statistical Association, Vol. 62, No. 317, March 1967, pp. 48-62.

3.0 Purpose: To provide Bayes estimates of: Mean time to failure, and probability of survival R(t). Bayes confidence intervals are also available but not reported here.

4.0 Limitations of Method

4.1 Serious limitations

4.1.1 Discovery of the form and parameters of the prior distribution.

4.2 Minor limitations

4.2.1 Sensitivity of method to assumptions on prior distributions not given.

5.0 Criteria Used in Measuring Efficiency of Method

No claims made, however, if assumptions hold the results are the correct model.

6.0 Knowledge Required to Use Method

6.1 The parameters and form of the prior must be known.

6.2 Test results must be available. No means of obtaining test time is given.

6.3 The failure times must be exponential.

7.0 Instructions for Use

7.1 Definitions for Table 5.1.5.

$$\gamma(n,x) = \int_0^x e^{-t}\, t^{n-1}\, dt$$

$$\gamma^*(n,y) = \gamma(n, \frac{y}{\alpha}) - \gamma(n, \frac{y}{\beta})$$

T_r = total time lived by all n items on test.

r = number of observed failures.

t = mission time.

$K_j(Z)$ = modified Bessel function of the third kind of order j.

$K_{-j}(Z) = K_j(Z)$

7.2 Select form of prior distribution and obtain estimates of all parameters in it.

7.3 Analyze test results to find r and T_r.

7.4 Apply formulae given in Table 5.1.5.

7.5 If confidence limits are desired, original paper must be consulted for the posterior distributions.

8.0 Advantages of Method

Under the assumptions, it is the correct model.

9.0 Reported Experince with Method: None.

TABLE 5.1.5

Prior Distribution on θ	$\hat{\theta}$	$\hat{R(t)}$
$g(\theta) = \frac{(a-1)\ (\alpha\beta)^{a-1}}{\beta^{a-1} - \alpha^{a-1}} \frac{1}{\theta^{a}}$ $(a < \alpha \le \theta \le \beta)$	$\frac{\gamma(r+a-2, T_r/\alpha) - \gamma(r+a-2, T_r/\beta)}{\gamma(r+a-1, T_r/\alpha) - \gamma(r+a-1, T_r/\beta)}$	$\frac{\gamma^*(r-1, T_r+t)}{\gamma^*(r-1, T_r)} \cdot \frac{1}{(1+t/T_r)^{r-1}}$ for $\underline{\underline{a = 0}}$ only
$g(\theta) = 1/\lambda\ e^{-\theta/\lambda}$ $(0 < \theta < \infty,\ \lambda > 0)$	$\sqrt{\lambda T_r}\ \frac{K_{r-2}\left(2\sqrt{\frac{T_r}{\lambda}}\right)}{K_{r-1}\left(2\sqrt{\frac{T_r}{\lambda}}\right)}$	$\frac{K_{r-1}\left(2\sqrt{\frac{T_r+t}{\lambda}}\right)}{K_{r-1}\left(2\sqrt{\frac{T_r}{\lambda}}\right)} \cdot \frac{1}{\left(1+\frac{t}{T_r}\right)^{\frac{(r-1)}{2}}}$
$g(\theta) = \frac{e^{-\mu/\theta}\left(\frac{\mu}{\theta}\right)^{\nu+1}}{\mu\Gamma(\nu)}$ $(0 < \theta < \infty,\ \mu,\ \nu > 0)$	$\frac{\mu + T_r}{r+\nu-1}$, $(r+\nu > 1)$	$\frac{1}{\left(1 + \frac{t}{T_r+\mu}\right)^{r+\nu}}$

5.1.6

1.0 Title of Method (Author's Title): A New Reliability Assessment Technique, A. Pozner.

2.0 Source: Technical Conference Transactions, American Society for Quality Control, 1966, pp. 188-201.

3.0 Purpose: To combine prior estimate of probability of success into a total estimate of probability of success (Bayes estimate).

4.0 Limitations of Method

4.1 Serious limitations

4.1.1 How discover form of prior is beta and if so, how discover values of a, b.

4.1.2 Method good only for beta prior.

4.1.3 No estimate given of sensitivity to wrong values of a, b or if beta is not the form of the prior.

5.0 Criteria Used in Measuring Efficiency of Method

I general, none. In particular, an example is given in which the Bayes estimate risk is smaller than the classical estimat. risk.

6.0 Knowledge Required to Use Method

6.1 Must know the two parameters say a, b of the prior beta distribution:

$$g(\theta) = \frac{\Gamma(b)}{\Gamma(a)\Gamma(b-a)} \theta^{a-1} (1-\theta)^{b-a-1} \quad 0 < \theta < 1,\ a > 0,\ b > a$$

6.2 It must be known that the successes (in n trials) occur from a Bernoulli trial situation, e.g., binomial type distribution.

6.3 The number of observed successes/trials must be known.

7.0 Instructions for Use

7.1 Determine a, b in the prior.

7.2 Find the observed successes, say A, in B total trials.

7.3 Estimate probability of success as

$$\frac{a + A}{b + B}$$

8.0 Advantages of Method

8.1 If the prior is correct, i.e., a,b are correct and the prior is the beta distribution, this method is the correct method as against the classical method of using just $\frac{A}{B}$.

9.0 Reported Experience with Method.

Examples given of use but no experience reported.

5.1.7

1.0 Title of Method (Author's Title): Optimum Decision Criteria for Reliability Test, S. A. Greenberg and I. Bosinoff.

2.0 Source: Mitre Corporation, May 1967.

3.0 Purpose: To select number of items to test, number of failures to observe and the acceptance criterion.

4.0 Limitations of Method

4.1 Serious limitations

4.1.1 Assumption of a two point discrete prior seems unlikely to hold.

4.1.2 How to discover p, C, C_o, C_1 not given.

4.2 Minor limitations

4.2.1 No instructions given on how far to extend the calculations in r.

5.0 Criteria Used in Measuring Efficiency of Method

The test selected minimizes the expected cost (risk) of wrong decision.

6.0 Knowledge Required to Use Method

6.1 Failure times must be known to be exponential.

6.2 θ_o and θ_1 (specified and min. acceptable MTBF's in the sense of MIL-STD-781A) must be selected.

6.3 The prior probability of θ_o, i.e., $P(\theta_o)$ must be known (say p) and also the prior probability of θ_1, i.e., $P(\theta_1)=1-p$. That is, a two point discrete prior is assumed and p must be known.

6.4 The following costs must be known

6.4.1 Cost of test (per failure) called C.

6.4.2 C_o = cost of rejecting an item with MTBF = θ_o.

6.4.3 C_1 = cost of accepting an item with MTBF = θ_1.

7.0 Instructions for Use

7.1 Select θ_o, θ_1, C, C_o, C_1, p (see Section 6).

7.2 Find a $\hat{\theta}_D$ for each r from (no limits given on how far to carry r).

$$\hat{\theta}_D = \frac{\left[\log_e \frac{\theta_o}{\theta_1} - \frac{1}{r} \log_e\left(\frac{pC_o}{(1-p)C_1}\right)\right]}{\left(\frac{1}{\theta_1} - \frac{1}{\theta_o}\right)} \qquad \theta_o > \theta_1$$

7.3 Prepare a table of pairs $(r, \hat{\theta}_D)$ and for each pair compute the average risk.

$$\bar{C}(\hat{\theta}_D) = pC_o \int_{\infty}^{\hat{\theta}_D} f_o(\hat{\theta})d\hat{\theta} + (1-p)C_1 \int_{\hat{\theta}_D}^{\infty} f_1(\hat{\theta})d\hat{\theta}$$

where $f_o(\hat{\theta})$, $f_1(\hat{\theta})$ are the Gamma sampling distributions of $\hat{\theta}$ under the hypothesis θ_o and θ_1 respectively.

7.4 For each triple $(r, \hat{\theta}_D, \bar{C})$ add to $\bar{C}$ the cost/failure multiplied by r to obtain total average risk.

7.5 Select the $r, \hat{\theta}_D$ with the smallest total average risk and test r items to failure; if

$$\hat{\theta} = \frac{t_1 + \text{----} + t_r}{r} \geq \hat{\theta}_D \text{ accept}$$

otherwise reject.

8.0 Advantages of Method

Minimizes expected costs of wrong decision.

9.0 Reported Experience with Method: None.

5.1.8

1.0 Title of Method (Author's Title): Interval Estimate of Reliability; Exponential Time to Failure, Gamma Prior Distribution on Failure Rate. C. W. Hamilton

2.0 Source: Bayesian Procedures and Reliability Information, I EE Proceedings Reliability I C.

3.0 Purpose: This method makes point and interval estimates of Reliability. R = Pr (item successfully completes a mission of length t). A point estimate is say $\tilde{R}$. An example of an interval estimate is $C_{R_o} = Pr(R > R_c)$.

4.0 Limitations of Method

Sufficient data must be available to define the prior distribution as Gamma.

5.0 Criteria Used in Measuring Efficiency of Method

This method is more powerful because it is possible to obtain $Pr(R > R_o)$ as an interval estimate instead of the usual confidence statement, C_α, R is contained in the interval $(R_o, 1)$ with confidence (α) based on test results.

6.0 Knowledge Required to Use the Method

6.1 There must be sufficient prior data available to determine the parameters of the prior Gamma distribution. Since the Gamma is a two parameter distribution, two moments are a minimum.

6.2 There must be evidence that the time to failure distribution is exponential.

7.0 Instructions for Use

Step 1 - Convert information into Gamma Prior distribution on failure rate, λ. i.e., determine a and b for

$$f(\lambda) = \frac{b^a}{\Gamma(a)} \lambda^{a-1} e^{-b\lambda}$$

The maximum entropy solution for a and b is given in Method 5.1.2. a and b can also be obtained from the mean and variance. i.e., $E(\lambda) = a/b$; $\sigma^2(\lambda) = \frac{a}{b^2}$

Step 2 - Obtain the kernel of the reliability function $k(R)$

$$R = e^{-\lambda t} \text{ or } \lambda = -\frac{\ln R}{t} \qquad d\lambda = -\frac{dR}{R \cdot t}$$

Thus $k(R) = (-\ln R)^{a-1} \; R^{b-1}$

Step 3 - Use the kernel of likelihood function of the test results. i.e., T test hours and r failures gives reliability R

$$k_r(T|R) = (-\ln R)^r R^{T/t}$$

Step 4 - Uses Bayes theorem to determine the distribution of $(R|T)$

$$f_r(R|T) = \frac{1}{\Gamma(r+a)} \left(\frac{T}{t} + b\right)^{r+a} (-\ln R)^{r+a} \, R \left(\frac{T}{t} + b\right)$$

Step 5 - Compute C_{R_o} from

$$C_{R_o} = \chi^2\left[2\left(\frac{T}{t} + b\right)(-\ln R_o)\right]$$

where χ^2 has $2(r+a)$ degrees of freedom

8.0 Advantages of Method: (See Section 5)

9.0 Reported Experience with Method: None

5.1.9

1.0 Title: Point and Interval Estimates for Reliability Based on a Beta Prior Distribution of Reliability and Attribute Reliability Test Results.

2.0 Source: Breipohl, A. M., Prairie, R. R., and Zimmer, W. J. A Consideration of the Bayesian Approach in Reliability Evaluation, IEEE Transactions of Reliability, October 1965, pp. 107-113.

3.0 Purpose: To provide the point estimates $\hat{R}$ and R^* and Interval estimates $[R_u, R_L]$ $(1 - \alpha)$ based on reliability attribute testing of an item. $\hat{R}$ is the mode of the posterior distribution. R^* is the mean of the posterior distribution.

4.0 Limitations of Method

4.1 It requires prior knowledge to determine i and j the parameters of the prior.

4.2 The method is inferior to parametric methods if the mission time is long or continuous since much information is lost in attribute sampling. The method is useful on, say, missile tests but inferior to other methods for radars or computers.

4.3 The method must be applied to items having a known mission time since reliability is estimated.

5.0 Criteria Used in Measuring Efficiency of Method

Sample size n required to produce a given expected width $E(R_u - R_L)$ is the criteria. This method reduces n beyond what would be required without the use of the Bayesian approach to produce the same value of $E(R_u - R_L)$.

6.0 Knowledge Required to Use Method

6.1 The prior distribution is known to be Beta.

6.2 The parameters, i and j, of the prior Beta distribution.

7.0 Instructions for Use

Step 1 - Collect reliability data on item. i.e., number of successes, m, in n trials.

Step 2 - Estimate selected statistic using

$$\hat{R} = \frac{i + m}{i+j+n}$$

$$R^* = \frac{m+i+1}{n+i+j+2}$$

$[R_u, R_L]$ $(1 - \alpha)$ from

R_u is the solution to $I_{R_u}\left[(m+i),(j+n-m)\right] = 1 - \alpha/2$

R_L is the solution to $I_{R_L}\left[(m+i),(j+n-m)\right] = \alpha/2$

where: i and j are the parameters of the Beta prior for R.

$$f(r;i,j) = \frac{\Gamma(i+j+2)}{\Gamma(i+1)\,\Gamma(j+2)}\, r^i\,(1-r)^j$$

$\Gamma(x)$ is the gamma function

$I_x(a,b)$ is the incomplete Beta function tabled in Tables of the Incomplete Beat Function, K. Pearson, Cambridge, England, University Press, 1934.

m - number of successes

n - number of trials.

8.0 Advantages of Method

8.1 It allows estimation of points and intervals from attribute (success or failure) data.

8.2 It requires fewer samples to reduce $E(R_u - R_L)$ to a desired width.

9.0 Reported Experience with Method: None.

5.1.10

1.0 Title: Reliability Demonstration Test Plans Using a Gamma Prior Distribution for Failure Rate and Posterior Producer's and Consumer's Risks.

2.0 Source: R. E. Schafer, Bayes Reliability Demonstration Test Plans, Hughes Aircraft Co., Fullerton, California.

3.0 Purpose: To determine the sample test times, T, and the admissible number of failures, c, for a demonstration of failure rate λ such that the posterior consumer's risk $(P(\lambda > \lambda_1 | A))$ is smaller than β^* and the posterior producer's risk $(P(\lambda < \lambda_0 | R))$ is smaller than α^*. A is acceptance and R is rejection of the reliability of the item.

4.0 Limitations of Method

4.1 Knowledge is required as per (6) below

5.0 Criteria Used in Measuring Efficiency of Method

The sample test time T required for the test.

6.0 Knowledge Required to Use Method

6.1 Time to failure is exponentially distributed (so that λ is independent of time on test).

6.2 α^*, β^*, λ_1, λ_0

6.3 Parameters v and α of the prior gamma distribution on λ,

$$p(\lambda) = \frac{\alpha^v}{\Gamma(v)} \lambda^{v-1} e^{-\alpha\lambda}$$

7.0 Instructions for Use

To derive the test plans, find the smallest T such that

$$\beta^* \geq P(\lambda > \lambda_1 | A) = \frac{\sum_{x=0}^{c}\left[\int_{\lambda_1}^{\infty} \frac{\alpha^v}{\Gamma(v)} \frac{\lambda^{v+x-1}}{x!} T^x e^{-\lambda(\alpha+T)} d\lambda\right]}{\sum_{x=0}^{c} \frac{\Gamma(v+x)}{\Gamma(v)x!} p^v q^x}$$

$$p = \frac{\alpha}{\alpha+T} \qquad q = \frac{T}{\alpha+T} \tag{1}$$

and

$$\alpha^* \geq P(\lambda < \lambda_o | R) = \frac{\sum_{x=c+1}^{\infty}\left[\int_{0}^{\lambda_o} \frac{\alpha^v}{\Gamma(v)} \frac{\lambda^{v+x-1}}{x!} T^x e^{-\lambda(\alpha+T)} d\lambda\right]}{\sum_{x=c+1}^{\infty} \frac{\Gamma(v+x)}{\Gamma(v)x!} p^v q^x} \tag{2}$$

The author's instructions for finding T and c are:

Step 1 - Choose T=25 c=0 and see if Equation (1) is less than or equal to β^*. If not, proceed to T=50 c=0. Proceeding by increments of 25 time units, find the smallest T, say T^* for which Equation (1) is $\leq \beta^*$. Try this combination, i.e. (c=0,T^*) in Equation (2). If Equation (2) $\leq \alpha^*$ the job is done. If not, proceed to Step 2.

Step 2 - Choose T=25 c=1 and see if Equation (1) $\leq \beta^*$. If not, proceed through T in steps of 25 units until finding a (T^*,c=1) which causes Equation (1) $\leq \beta^*$. Try (T^*, c=1) in Equation (2) and see if Equation (2) $\leq \alpha^*$, if so, the job is done. If not, choose T=25 c=2 and repeat process. It will terminate.

8.0 Advantages of Method

8.1 It considers the posterior risk which is usually the most meaningful risk indicator.

8.2 The test time T is shorter than conventional fixed time tests.

9.0 Reported Experience with Method: None.

5.1.11

1.0 Title of Method (Author's Title): A Bayesian Approach for Designing Component Life Tests, Harold S. Balaban.

2.0 Source: Proceedings of the National Symposium on Reliability and Quality IEEE, 1967, pp. 59-74.

3.0 Purpose: Consider a system of n components each with life times distributed exponentially and independently. This paper gives a method of selecting the component test times T_i and the acceptance numbers r_i to satisfy a specified probability (β_S) that the system failure rate, on acceptance, is greater than a preselected λ_S^*. Usually, β_S is small so that λ_S^* is something like maximum acceptable failure rate (in the sense of MIL-STD-781A, it is the reciprocal of θ_1).

4.0 Limitations of Method

4.1 Serious limitations

4.1.1 How to discover form and parameters of prior distribution.

4.1.2 For the only continuous prior given (i.e., Gamma) it was assumed all n component's priors had the same scale parameter b. This is highly unlikely.

4.1.3 The equations for discrete prior distributions are not given with respect to form (e.g., binomial prior, Polyá prior).

4.2 Minor limitations

4.2.1 The author equates (see 7.3) the classical consumer's risk β:

$$\beta = P(\text{acceptance} \mid \text{unacceptable failure rate})$$

with his β_S:

$$\beta_S = P(\text{failure rate greater than or equal to unacceptable failure rate} \mid \text{acceptance})$$

5.0 Criteria Used in Measuring the Efficiency of Method

None given but generally if the assumptions are correct and if the prior distribution is "good" test time will be less than for the classical methods.

6.0 Knowledge Required to Use Method

6.1 λ_S^* must be selected.

6.2 β_S must be selected.

6.3 α_i (for each component) must be selected, i.e., the producer's risks.

6.4 A prior distribution must be selected (with all its parameters known).

7.0 Instructions for Use

7.1 Determine prior distribution (prior distributions limited to discrete or Gamma).

7.2 Determine λ_S^* and allocate it (method generally unspecified) to the component level so that $\lambda_1 + \lambda_2 --- + \lambda_n = \lambda_S^*$

7.3 Determine β_S. Here the author takes β_S as the classical β:

$P(\text{Acceptance}|\lambda_S = \lambda_S^*) = \beta$(in the sense of MIL-STD-781A) $= \beta_S$
but β_S is really
$P(\lambda_S \geq \lambda_S^*|\text{Acceptance})$

Unless it is desired that

$P(\text{Acceptance}|\lambda_S = \lambda_S^*) = P(\lambda_S \geq \lambda_S^*|\text{Acceptance})$ there is no reason to select $\beta = \beta_S$.

7.4 For an n component system (all failing independently and exponentially for given λ) with each of the n prior Gammas all with the same scale parameter (b) solve the equation

$$1-\beta_S = \int_0^{\lambda_S^*} \frac{e^{-y/\mu}\, y^{\sum^n a_i - 1}}{\Gamma\left(\sum^n a_i\right) \mu^{\sum a_i}} dy,$$

where: a_i is the shape parameter of the i^{th} prior Gamma and

$\mu = \dfrac{b}{1+bT}$, (b the common scale parameter of the prior Gammas)

for T. Select the component producer's risks, i.e., the α_i.

7.5 Taking the T of 7.4 and assuming the acceptance number of each component test is zero, i.e., $r_i=0$, find the classical β risk for each component test (using the previously obtained allocated component failure rates, Section 6.2).

7.6 Now, using the n classical β's developed in 7.5 and the previously selected n α_i's and making the assumption that each of the n acceptable failure rates is 1/2 the allocated failure rates of 7.2 design n component tests. The result will be n pairs (T_i, n_i).

7.7 On basis of what was accomplished in 7.1-7.6 the (posterior) probability that $\lambda_S \geq \lambda_S^*$ is $1-\beta_S$ provided all components pass their tests.

N.B. Formulae are given in the paper for a discrete prior.

8.0 Advantages of Method

8.1 Provides an unambiguous method of designing component life tests in the face of a system requirement.

8.2 Provides for incorporation of prior failure rate knowledge into the decision procedure.

8.3 If the prior distribution is "good" test time may be saved.

9.0 Reported Experience with Method: None.

5.1.12

1.0 Title of Method (Author's Title): Bayesian Reliability Demonstration Plans, Austin J. Bonis.

2.0 Source: Conference Proceedings, 5th Annals of Reliability and Maintainability AIAA, 1966, pp. 861-873.

3.0 Purpose: To calculate probability that an "accepted" system has an acceptable failure rate.

4.0 Limitations of Method

4.1 Serious limitations.

4.1.1 The assumption that the prior distribution of failure rate is discrete with only two points (λ_1 and λ_2), i.e., that $p + q = 1$ is not only not verified by historical data but highly unlikely to hold in the real world.

4.1.2 Does not solve the real problem: How to select (T,r) so that a given confidence is achieved. It is then an "after the fact" method of data analysis.

4.2 Minor limitations

4.2.1 The validity of the results depends on the accuracy of estimating p and q.

5.0 Criteria Used in Measuring Efficiency of Method.

After test is complete, higher confidence levels (for failure rate) are usually achieved than the classical methods.

6.0 Required Knowledge to Use Method

6.1 Two values of failure rate λ_1, λ_2 such that λ_1 is "good" and λ_2 is "bad" ($\lambda_1 < \lambda_2$).

6.2 Two probabilities p and q such that the prior probability of occurrence of λ_1 is p and the prior probability of occurrence of λ_2 is q; $p + q = 1$.

6.3 Test time T, evidently arrived at by engineering means, i.e., T is called "available" test time.

6.4 Times to failure are exponential.

7.0 Instructions for Use

7.1 Select λ_1 (usually specified failure rate in the sense of MIL-STD-781A, i.e., $1/\theta_0 = \lambda_1$).

Select λ_2 (usually maximum acceptable failure rate in the sense of MIL-STD-781A, i.e., $1/\theta_1 = \lambda_2$).

7.2 Estimate p, q from previous engineering history/experience.

7.3 Select T from test time available.

7.4 After test is over, record number of failures observed say r.

7.5 Compute the quantity

$$P(\lambda_1|r) = \frac{pe^{-\lambda_1 T}(\lambda_1 T)^r}{pe^{-\lambda_1 T}(\lambda_1 T)^r + qe^{-\lambda_2 T}(\lambda_2 T)^r}$$

this is the probability the system possesses failure rate λ_1. $1-P(\lambda_1|r)$ is the probability the system possesses failure rate λ_2.

8.0 Advantages of Method

8.1 Generally will give higher confidence than classical methods.

8.2 If assumptions hold, it is the correct model.

9.0 Reported Experience with Method

None.

5.2 Distribution Dependent Methods.

1.0 Title of Method (Author's Title): Estimation of the Shape and Scale Parameters of the Weibull Distribution, M. V. Menon.

2.0 Source: Technometrics, Volume 5, No. 2, May 1963.

3.0 Purpose: To evaluate estimators, $\hat{b}$ and $\hat{c}$, of the scale parameter, b, and the shape parameter, c, of the Weibull distribution

$$f(x) = (c/b)(x/b)^{c-1} \exp(-(x/b)^c)$$

for the cases (i) b and c unknown (ii) b known. The case where c is known is covered in Method 5.2.3.

4.0 Limitations of Method

4.1 The estimators except in the unusual case when b is known are not very efficient. The asymptotical efficiency of $\hat{c}$ with b unknown is 55%. The efficiency of $\hat{b}$ is not given.

4.2 $\hat{c}$ is less efficient than c* of Method 5.2.2.

5.0 Criteria Used in Measuring Efficiency of Method

Characteristics of the estimators and mathematical tractability are used to measure efficiency of the method.

6.0 Knowledge Required to Use Method

n sample results from the Weibull population $(x_1, x_2, \ldots x_n)$

7.0 Instructions for Use

7.1 Instructions are only given for b and c unknown since b known is very unusual.

7.2.1 Compute the values $\ln x_i$ for $i=1,2, \ldots n$

7.2.2 Compute $\hat{d}$ from

$$\hat{d} = \left\{\frac{6}{\pi^2}\left[\sum_{i=1}^{n} (\ln x_i)^2 - \left(\sum_{i=1}^{n} \ln x_i\right)^2/n\right]/(n-1)\right\}^{1/2}$$

7.2.3 Compute $\hat{c}$ from

$$\hat{c} = \frac{1}{\hat{d}}$$

7.2.4 Compute $\ln\hat{b}$ from

$$\ln\hat{b} = \left(\sum_{i=1}^{n} \ln x_i\right)/n + .5772\,\hat{a}$$

7.2.5 Compute $\hat{b}$ from

$$\hat{b} = \exp(\ln\hat{b})$$

7.2.6 $\hat{b}$ and $\hat{c}$ are asymptotically normal with means and variances given by

$$E(\hat{c}) = c$$

$$\mathrm{Var}(\hat{c}) = 1.1c^2/n$$

$$E(\hat{b}) = b$$

$$\mathrm{Var}(\hat{b}) = 1.2b^2/(nc^2)$$

8.0 Advantages of Method

8.1 It is the only analytic method that uses all the data from a sample of size $n > 2$ for estimating b and c when both are unknown.

8.2 The estimators are asymptotically normal and unbaised.

8.3 The estimators are easy to compute.

8.4 $\hat{b}$ is a more efficient estimator than b* of Method 5.2.2.

9.0 Reported Experience with Method: Not applicable.

5.2.2

1.0 Title of Method (Author's Title): Some Percentile Estimators for Weibull Parameters, S. D. Dubey.

2.0 Source: Technometrics, Volume 9, No. 1, February 1967.

3.0 Purpose: The method provides estimators b* and c* for the parameters b and c of a Weibull density

$$f(x) = (c/b)(x/b)^{c-1} \exp(-(x/b)^c)$$

and m* and θ* for the parameter m and θ from the Weibull density

$$f(x) = (m/\theta)x^{m-1} \exp(-x^m/\theta)$$

using the order statistics corresponding to particular percentiles of the sample results.

4.0 Limitations of Method

4.1 The efficiency of c* is not great. Asymtotic efficiency is 66% relative to the maximum likelihood estimator of c.

4.2 The estimator $\hat{b}$ of Method 5.2.1 is more efficient than b*.

5.0 Criteria Used in Measuring Efficiency of the Method

Characteristics of the estimators and computational ease of making the estimates.

6.0 Knowledge Required to Use the Method

The values of the order statistics y_{c1}, y_{c2} or y_{b1}, y_{b2} or y_{j1}, y_{j2}

7.0 Instructions for Use of the Method

7.1 Determine the order statistics to use from

y_{c1} is the order statistic such that c1 is the smallest interger $\geq .1637n$

y_{c2} is the order statistic such that c2 is the smallest interger $\geq .9737n$

y_{b1} is the order statistic such that b1 is the smallest interger $\geq .3978n$

y_{b2} is the order statistic such that b2 is the smallest integer $\geq .8211n$

y_{j1} is the order statistic such that J1 is the smallest integer $\geq .2338n$

y_{j2} is the order statistic such that J2 is the smallest integer $\geq .9266n$

7.2 Compute

$$p_1 = c1/n$$

$$p_2 = c2/n$$

$$p_3 = b1/n$$

$$p_4 = b2/n$$

$$p_5 = J1/n$$

$$p_6 = J2/n$$

7.3 Compute c^* or m^* from

$$c^* = m^* = k/(\ln y_{c1} - \ln y_{c2})$$

where $k = \ln(-\ln(1-p_1))-\ln(-\ln(1-p_2))$

7.4 Compute b^* from

$$b^* = \exp\left[1/2\sum_{i=1}^{2}\left\{\ln y_{bi} - \ln(-\ln(1-p_{i+2}))/\tilde{c}\right\}\right]$$

where $\tilde{c} = \left[\ln(-\ln(1-p_3))-\ln(-\ln(1-p_4))\right]/(\ln y_{b1}-\ln y_{b2})$

7.5 Compute θ^* from

$$\theta^* = y_{b1}^{\tilde{c}}/\ln(1-p_3)$$

7.6 Compute b^*, c^* or m^*, θ^* jointly using y_{j1}, y_{j2} and p_5, p_6 in 7.3, 7.4 or 7.3, 7.5.

7.7 The estimators are all asymptotically unbiased and normal.

8.0 Advantages of Method

8.1 Estimates can be made with a minimum of data (i.e., 2 order statistics).

8.2 c^* using y_{c1} and y_{c2} is a more efficient estimate than $\hat{c}$ of Method 5.2.1.

8.3 The estimates are asymptotically unbiased and normal.

8.4 The computation of values is easy.

9.0 Reported Experience with Method: Not applicable.

5.2.3

1.0 Title of Method (Author's Title): Point and Interval Estimators Based on m Order Statistics for the Scale Parameter of a Weibull Population with Known Shape Parameter, H. L. Harter and A. H. Moore.

2.0 Source: Technometrics, Volume 7, No. 3, August 1965.

3.0 Purpose: To supply (1) the maximum likelihood and an unbiased point estimate and (2) the 1-2P confidence interval estimate for the scale parameter, θ, when the values of the first m order statistics are available from the Weibull population with known shape parameter K given below

$$f(y) = (K/\theta)(y/\theta)^{K-1} \exp(-(y/\theta)^K)$$

4.0 Limitations of Method

4.1 The shape parameter, K, must be known.

4.2 Further computations are necessary because the scale parameter alone does not have a physical interpretation.

5.0 Criteria Used in Measuring Efficiency of Method

Characteristics of the estimators and mathematical tractability.

6.0 Knowledge Required to Use the Method

6.1 The value of the shape parameter K.

6.2 The sample size n.

6.3 The values of the first m order statistics $y_1, y_2, y_3, \ldots y_m$.

7.0 Instructions for Use

7.1 Compute $x_1, x_2, \ldots x_m$ from $x_i = y_i^K$

7.2 Compute the maximum likelihood estimator of θ, $\hat{\theta}_{mn}$ from

$$\hat{\theta}_{mn} = \left\{ \left[x_1 + x_2 + \ldots x_{m-1} + (n-m)x_m \right] / m \right\}^{1/K}$$

7.3 Compute the unbiased estimator of θ, $\tilde{\theta}$ from

$$\tilde{\theta} = \left(\frac{\tilde{\theta}}{\hat{\theta}}\right)\hat{\theta}_{mn}$$

where $\left(\frac{\tilde{\theta}}{\hat{\theta}}\right)$ is found from the table in the source document at the appropriate n and m.

7.4 Compute the (1-2P) confidence interval and points $\bar{\theta}_{(1-p)}$ and $\underline{\theta}_p$ from

$$\underline{\theta}_p = \left(2m/\chi^2_{2m,p}\right)^{1/K}\hat{\theta}_{mn}$$

$$\bar{\theta}_{1-p} = \left(2m/\chi^2_{2m,1-p}\right)^{1/K}\hat{\theta}_{mn}$$

where $\chi^2_{2m,p}$ is the value of chi square for 2m degrees of freedom that has p probability of being exceeded.

8.0 Advantages of Method

It is an efficient and sufficient method of estimating θ.

9.0 Reported Experience with Method: Not applicable.

5.2.4

1.0 Title of Method (Author's Title): Estimation of Weibull Distribution Shape Parameter When No More Than Two Failures Occur Per Lot, J. L. Jaech.

2.0 Source: Technometrics, Volume 6, No. 4, November 1964.

3.0 Purpose: This method provides point and interval estimates of the shape parameter, a, of a Weibull distribution with density

$$f(x)=(a/b)x^{a-1}\exp(-x^{a}/b)$$

4.0 Limitations of Method

4.1 The characteristics (bias, efficiency, etc.) of the estimates are not given.

4.2 Additional failure times cannot be used to improve the estimates.

4.3 Trial and error iterative methods are needed for some estimates.

5.0 Criteria Used in Measuring Efficiency of Method

Characteristics of the estimators and mathematical tractability.

6.0 Knowledge Required to Use the Method.

The only knowledge required are the experimental results and the desired confidence; sample size (n), time of first failure (x_1), time of second failure (x_2), and confidence ($1-2\alpha$).

7.0 Instructions for Use

7.1 For a single experiment with n items and two failures at times x_1 and x_2.

7.1.1 Form $u = x_2/x_1$.

7.1.2 Compute the lower limit on a, a_L from

$$a_L = \frac{\ln\left\{(n-1+\alpha)/\left[(1-\alpha)(n-1)\right]\right\}}{\ln u}$$

which reduces to

$$a_L = \frac{-\ln(1-\alpha)}{\ln u} \qquad \text{for large } n.$$

7.1.3 Compute the upper limit on a, a_T from

$$a_T = \frac{\ln\left\{(n-\alpha)/\left[\alpha(n-1)\right]\right\}}{\ln u}$$

which reduces to

$$a_T = \frac{-\ln\alpha}{\ln u} \quad \text{for large n}$$

7.1.4 Compute the point estimate of a, $\tilde{a}$ by solving

$2 = -2\ln\left\{n/\left[1+(n-1)u^{\tilde{a}}\right]\right\}$ for $\tilde{a}$. For large n, $\tilde{a}$ is given by

$$\tilde{a} = \frac{1}{\ln u}$$

7.2 For k identical experiments with the same sample size n.

7.2.1 Compute $u_i = \frac{x_{2,i}}{x_{1,i}}$

7.2.2 Solve the following equation to estimate a_L

$$-2\sum_{i=1}^{k}\ln\left\{1-n/\left[1+(n-1)u_i^{a_L}\right]\right\} = \chi^2_{2k,\alpha}$$

where $\chi^2_{2k,\alpha}$ is the value of the Chi square distribution with 2k degrees of freedom which has probability α of being exceeded.

For large n solve

$$-2\sum_{i=1}^{k}\ln\left(1-u_i^{a_L}\right) = \chi^2_{2k,\alpha}$$

for a_L. Trial and error must be used.

7.2.3 Solve the following to find a_T

$$-2\ln\left\{n/\left[1+(n-1)u_i^{a_T}\right]\right\} = \chi^2_{2k,\alpha}$$

For large n

$$a_T = \chi^2_{2k,\alpha}/\left[2\sum_{i=1}^{k}\ln u_i\right]$$

7.2.4 For large n a point estimate of a $\hat{a}$ is

$$\hat{a} = \frac{k}{\sum_{i=1}^{k} \ln u_i}$$

8.0 Advantages of Method

It gives mathematically tractable point and interval estimates of the shape parameter of a Weibull distribution without any knowledge of the scale parameter.

9.0 Reported Experience with Method

The method is used on some data in the source paper.

5.2.5

1.0 Title of Method (Author's Title): One Order Statistic Estimation of the Scale Parameter of Weibull Distributions, A H. Moore and H. L. Harter.

2.0 Source: IEEE Transactions on Reliability, October 1965.

3.0 Purpose: This method provides the means of estimating the scale parameter, θ, for a Weibull distribution

$$f(Y) = (K/\theta)(Y/\theta)^{K-1} \exp(-Y/\theta)^{K})$$

when the shape parameter K is known from a single order statistic.

4.0 Limitations of Method

4.1 The shape parameter must be known.

4.2 The method is inefficient if the values of other sample results are available. (A maximum likelihood estimator is mathematically tractable.)

5.0 Criteria Used in Measuring Efficiency of Method

The criteria used are characteristics of the estimator, ease of data collection, and simplicity of calculation.

6.0 Knowledge Required to Use the Method

6.1 The value of the shape parameter.

6.2 The value of an order statistic.

7.0 Instructions for Use

7.1 Look up $C_{N,M}$ in table I of the source document. N is the sample size and M is the order statistic which is being used.

7.2 Estimate θ from $\tilde{\theta} = C_{N,M} Y_{N,M}$

where $Y_{N,M}$ is the value of the m^{th} order statistic.

8.0 Advantages of Method

8.1 The estimator is unbiased.

8.2 The estimator is easy to compute for sample sizes up to 20.

8.3 Only one piece of data is needed.

8.4 The estimate is fairly efficient if only a few pieces of data are available, i.e., the test is terminated when only a few samples have failed.

9.0 Reported Experience with Method: Not applicable.

5.2.6

1.0 Title of Method (Author's Title): Interval Estimates of Some Percentiles of a Weibull Distribution with Known Shape Parameter Based on One Order Statistic, R. E. Schafer and J. M. Finkelstein.

2.0 Source: Hughes Aircraft Company, Fullerton, California 92634

3.0 Purpose

This method is used to (1) make interval estimates of percentiles, L_p,(i.e., determine the interval such that the probability it covers the true value of L_p is equal to $(1-\gamma)$) from available test data and (2) aid in designing a life test for finding the interval estimate. Tables are available for the 1st, 5th, 10th and 20th percentiles for each of the n order statistics from a sample of size n with parameters (i) confidence levels: $1 - \gamma = .80, .90, .95, .99$ (ii) shape parameter $\beta = 0.5, 1.0\ (1)\ 4.0$ and (iii) $N = 1\ (1)\ 10\ (5)\ 25$. The Weibull distribution used is $F(x)=1-\exp(-x^{\beta}/\alpha)$

4.0 Limitations of Method

4.1 It is necessary to know the shape parameter β of the Weibull distribution to use the method.

4.2 One order statistic interval estimates are inefficient (have greater widths) relative to estimates using more data.

5.0 Criteria Used in Measuring Efficiency of Method

5.1 The expected width, $E\ (U_{(1-\gamma/2)} - L_{\gamma/2})$ determines the efficiency of the method for analyzing existing data.

5.2 Sample size n and expected test duration are used in measuring efficiency of life test designs. (Expected duration is equal to the expectation of the i^{th} order statistic, Ey_i, since the test will end with the occurrence of the i^{th} order statistic.)

6.0 Knowledge Required to Use the Method

6.1 Shape parameter, β, of distribution.

6.2 Value of one order statistic.

6.3 Confidence limit desired.

6.4 **Maximum** interval width requirement (when designing life tests).

6.5 Life test limiting factor i.e., sample size or time.

7.0 Instructions for Use

7.1 To find the interval estimate for the p^{th} percentile L_p from available data of a sample of size n.

7.1.1 Determine order statistics that are available (values of some samples may have been lost or not collected).

7.1.2 Go to the appropriate table for γ and β and find the order statistic to use by searching the expected interval width EW_i/L_p column for the smallest value for which an order statistic is available.

7.1.3 Go to the appropriate $\beta = 1$ table for γ and find the interval end points multipliers in the $C_p\ U^{-1}$ and $C_p\ L^{-1}$ columns.

7.1.4 Compute the interval end points $L_{\gamma/2}$ and $U_{(1-\gamma/2)}$ from

$$L_{\gamma/2} = (C_p L^{-1})^{1/\beta} y_i$$

$$U_{(1-\gamma/2)} = (C_p U^{-1})^{1/\beta} y_i$$

where y_i is the value of the i^{th} order statistic.

7.2 To find a life test design with which to estimate the p^{th} percentile.

7.2.1 Examine the EW_i/L_p column of appropriate table to find the combinations of n and i that meet the EW_i/L_p requirement.

7.2.2 If sample size is the test limitation select the smallest n from the admissible set.

7.2.3 If test time is the critical test characteristic find the n,i combination from the set in 7.2.1 which has the smallest Ey_i/L_p.

8.0 Advantages of Method

8.1 The percentiles of the Weibull are important in analyzing the reliability of items exhibiting wearout characteristics ($\beta > 1$ means the distribution has an increasing failure rate).

8.2 The percentiles have a natural physical interpretation while the scale (α) and shape (β) parameter do not.

8.3 The computation of the interval end points is very easy.

8.4 Only the value of a single order statistic is required.

8.5 The efficiency of the estimate can be compared with maximum likelihood estimates.

8.6 The estimates are efficient when only a few sample points are available (i.e., if a test is ended after a few failures).

8.7 Life test designs can be compared with designs using maximum likelihood computations from the tables in the source.

9.0 Reported Experience with Method: Not applicable.

5.2.7

1.0 Title of Method: Evaluation of Percentiles and Weibull Slope Using Order Statistics.

2.0 Source: Appendix B to Statistical Investigation of the Fatigue Life of Deep Groove Ball Bearings, J. Lieblein and M. Zelen, Journal of Research of the National Bureau of Standards, Vol. 57, No. 5, Nov. 1956. Also "A New Method of Analyzing Extreme Value Data; J. Lieblein, National Advisory Committee for Aeronautics, Tech. Note 3053.

3.0 Purpose: To estimate the Percentiles L_{10} and L_{50} and the Weibull slope of a Weibull Distribution of a part from life test data. Specifically the 90th percentile, the median and the scale parameter of the Weibull Distribution can be estimated. The Weibull CDF is

$$F(L)=1-\exp\left[-(L/a)^{e}\right]$$

4.0 Limitations of Method

4.1 Tables required for the method are not available for sample sizes greater than 6. The authors recommend a procedure if the sample size is greater than 6 but the authors say their scheme is not desirable if it can be avoided. (Note: N. Mann of Rocketdyne, Division of N.A.A. reports more extensive tables in an unpublished doctoral thesis.)

4.2 The efficiency of the method is unknown for sample sizes greater than 6.

5.0 Criteria Used in Measuring Efficiency of Method

5.1 The variance of estimators of the percentiles relative to the Cramer-Rao lower bound and ease of computation.

6.0 Knowledge Required to Use the Method

6.1 Sample size (n), number failing (k), times of failures $(L_1, L_2, \ldots L_k)$.

6.2 Belief that underlying distribution of lifetimes is Weibull.

7.0 Instructions for Use

7.1 For $n \leq 6$

7.1.1 Order the life (L_i) in ascending order smallest to largest

7.1.2 Compute $x_i = \ln (L_i)$

7.1.3 Compute $\hat{u}$ and $\hat{\beta}$ from

$$\hat{u} = \sum_{i=1}^{k} a_i x_i; \quad \hat{\beta} = \sum_{i=1}^{k} b_i x_i$$

where a_i and b_i are obtained from table B-2 of the reference. (Note: The a_i's are not the parameter of the CDF).

7.1.4 Compute L_{10}, L_{50}, and e from

$$L_{10} = \exp\left[\hat{u} - 2.25037\,\hat{\beta}\right]$$

$$L_{50} = \exp\left[\hat{u} - 0.36651\,\hat{\beta}\right]$$

$$e = 1/\hat{\beta}$$

7.2 $n > 6$

7.2.1 Make random selection of subgroups of size 6. This can be done by selection of random numbers to order the data points or by tagging the items into subgroups before the test starts.

7.2.2 Perform steps in 7.1.1 and 7.1.2 for each subgroup.

7.2.3 Compute $\bar{T}_1$ and $\bar{T}_2$ from

$$\bar{T}_1 = \frac{1}{Z}\sum_{j=1}^{Z} \hat{u}_j$$

$$\bar{T}_2 = \frac{1}{Z}\sum_{j=1}^{Z} \hat{\beta}_j$$

where Z is the number of subgroups and $\hat{u}_j$ and $\hat{\beta}_j$ are results of 7.2.2 for subgroup j.

7.2.4 Compute L_{10}, L_{50}, and e from

$$L_{10} = \exp\left[\bar{T}_1 - 2.25037\bar{T}_2\right]$$

$$L_{50} = \exp\left[\bar{T}_1 - 0.36651\bar{T}_2\right]$$

$$e = 1/\bar{T}_2$$

8.0 Advantages of Method

8.1 The method can be used when data is censored i.e., the total sample does not fail before the test ends.

8.2 The method accounts for dependency of successive data points.

8.3 The estimates are unbiased.

9.0 Reported Experience with Method

Some use with actual data is reported in the source.

5.2.8

1.0 Title of Method: Linear Invariant Estimation of the Percentiles of a Weibull Distribution.

2.0 Source: Tables for Obtaining the Best Linear Invariant Estimates of Parameters of the Weibull Distribution, Nancy R. Mann, Research Report 66-9, March 1966, Rocketdyne, Division of North American Aviation.

3.0 Purpose: To estimate the percentiles, $\tilde{T}_{(1-R)}$, of a random variable having a Weibull distribution such that the expected mean squared error is a minimum. This method is the same as 5.2.7 except that the estimators are allowed to be biased and the tables are more extensive.

4.0 Limitations of Method

4.1 The estimates for small samples are not generally unbiased.

4.2 The method for $n > 25$ is of unknown efficiency.

5.0 Criteria Used in Measuring Efficiency of Method

The characteristics of the estimator and the ease of computation were used.

6.0 Knowledge Required to Use the Method

6.1 The sample size n and the number of observations m ($m \leq n$).

6.2 The percentile of interest (1-R).

6.3 The values of the sample $t_1, t_2, t_3, \ldots t_m$.

7.0 Instructions for Use

7.1 Order the sample values from smallest to largest.

7.2 Compute $x_i = \ln t_i$.

7.3 Compute $\tilde{u}$ and $\tilde{b}$ from $\tilde{u} = \sum_{i=1}^{m} A_i x_i$; $\tilde{b} = \sum_{i=1}^{m} B_i x_i$

where m is the number of failures and the A_i and B_i are obtained from table I in the source document for appropriate n and m.

7.4 Compute $\tilde{T}_{(1-R)}$ from $\tilde{T}_{(1-R)} = \exp\left[\tilde{u} + \tilde{b} \ln \ln(1/R)\right]$

7.5 If $n > 25$, divide the sample randomly into k groups of size < 25.

7.5.1 Perform 7.1 through 7.3 for each subgroup.

7.5.2 Compute an overall $\tilde{u}$ and $\tilde{b}$ from

$$\tilde{u} = \sum_{j=1}^{k} w_j \tilde{u}_j \; ; \quad \tilde{b} = \sum_{j=1}^{k} w_j \tilde{b}_j$$

where $w_j = n_j/m$ (n_j is the number in subgroup j)

and $\tilde{u}_j$ and $\tilde{b}_j$ are the estimates for subgroup j.

7.5.3 Compute $\tilde{T}_{(1-R)}$ from the expression in 7.4 using $\tilde{u}$ and $\tilde{b}$ from 7.5.2.

8.0 **Advantages of Method**

8.1 The estimate has smaller mean squared error than the estimates from Method 5.2.7.

8.2 The method works for censored samples.

8.3 The estimators are asymptotically efficient and normal.

8.4 The estimates are easy to compute.

9.0 Reported Experience with Method: Not applicable.

5.2.9

1.0 Title of Method (Author's Title): A Technique for Estimating Weibull Percentile Points, J. S. White.

2.0 Source: Annals of Reliability and Maintainability, 5th Reliability and Maintainability Conference, Volume 5, 18-20, July 1966, American Institute of Aeronautics and Astronautics, New York, New York, 10019.

3.0 Purpose: To make a point estimate, $T(p)^*$, and interval estimates ($T_L(p)$ and $T_U(p)$) for the p^{th} percentile of a Weibull distribution, $F(T) = 1-\exp(-(T/\theta)^{\beta})$.

4.0 Limitations of Method

4.1 None of the properties of the estimator or the width of the confidence interval are given.

4.2 Computation of the intervals is only possible for the sample sizes covered in table VII in the source i.e., N=20, 30, 40, 50 70, 100, 500, 1000.

4.3 p is restricted to .01, .05, .10, .50, .95 and .99.

5.0 Criteria Used in Measuring Efficiency of Method

The characteristics of the estimators and the ease of computation we. used.

6.0 Knowledge Required to Use the Method

6.1 The sample results (T_1, T_2, ...T_N) from a sample of size N.

6.2 The confidence level C desired.

6.3 The percentile p to be estimated.

7.0 Instructions for Use

7.1 Compute X_i from $X_i = \ln T_i$

7.2 Compute $\bar{X}$ from $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$

7.3 Compute S from

$$S = \left[\frac{1}{N-1}\sum_{i=1}^{N}(X_i-\bar{X})^2\right]^{1/2}$$

7.4 Compute $X(p)^*$ from $X(p)^* = \overline{X} + \frac{Z(p)S}{B(N)}$

where $Z(p)$ and $B(N)$ are obtaindd from tables II and VI in the source documents.

7.5 Compute $T(p)^*$ from $T(p)^* = \exp(X(p)^*)$

7.6 Compute the C confidence level interval endpoints from

$$T_L(p) = \exp\left[X(p)^* - DS\right]$$

$$T_U(p) = \exp\left[X(p)^* + DS\right]$$

where D is obtained from table VII of the source document.

8.0 Advantages of Method

8.1 The percentiles of the Weibull have important reliability applications.

8.2 Estimates and confidence intervals can easily be computed.

9.0 Reported Experience with Method

A sample of the use of the method is given in the source document.

5.2.10

1.0 Title of Method (Author's Title): An Exact Asymptotically Efficient Confidence Bound for Reliability in the Case of the Weibull Distribution, M. V. Johns, G. J. Lieberman.

2.0 Source: Department of Statistics, Stanford University, Technical Report No. 75, December 1964.

3.0 Purpose: This method provides exact confidence bounds on reliability for time t_o, $R(t_o)$, when the times to failure follow the Weibull Distribution $F(t) = 1 - \exp(-(t/\alpha)^\beta)$, based on a sample of times to failure.

4.0 Limitations of Method

4.1 For sample size n less than 50 results are only available for $n = 10, 15, 20,$ and 30.

4.2 The efficiency for small n is unknown.

4.3 No knowledge regarding the shape parameter is used i.e., knowing that the shape parameter β is greater than or less than unity would lead to sharper bounds.

5.0 Criteria Used in Measuring Efficiency of Method

The characteristics of the confidence bound and the ease of computation are used.

6.0 Knowledge Required to Use the Method

6.1 The results of a life test.

6.1.1 The sample size n.

6.1.2 The number failed in the test, r.

6.1.3 The times of failure of failed items $X_{(1)}, X_{(2)}, \ldots X_{(r)}$.

6.2 The time of interest t_o.

6.3 The confidence desired γ.

7.0 Instructions for Use

7.1 Compute the set of $\{Y_{(i)}\}$ from $Y_{(i)} = \ln X_{(i)} - \ln t_o$

7.2 Compute Z_a from $Z_a = \sum_{i=1}^{r} a_i Y_{(i)}$

where the a_i's are obtained from table II in the source document.

7.3 Compute Z_b from $Z_b = \sum_{i=1}^{r} b_i Y_{(i)}$

where the b_i's are obtained from table II of the source document.

7.4 Compute Z_a/Z_b.

7.5 Enter table I at Z_a/Z_b, n, r, and γ and read the lower confidence bound $L^* (Z_a/Z_b)$.

7.6 For n > 50 and conditions not contained in table I equation 30 in the source document can be used.

7.6.1 Compute p from $p = r/n$

7.6.2 The values of α_1 (p) and β_2 (p) are contained in table III of the source document.

7.6.3 $K_{(1-\gamma)}$ is the value of normal distribution exceeded $(1-\gamma)\%$ of the time. Table 2 in Mathematical Methods of Statistics by H. Cramer can be conveniently used. $K_{(1-\gamma)}$ equivalent to $\lambda_{(\gamma/2)100}$ in the table.

8.0 **Advantages of Method**

8.1 The confidence bounds are asymptotically efficient.

8.2 The bounds are easy to compute using the tables.

8.3 No assumptions except that the distribution is Weibull are necessary.

9.0 **Reported Experience with Method:** Not applicable.

5.2.11

1.0 Title of Method: Improved Lower Confidence Bounds on the Reliability Function of a Weibull Distributed Random Variable.

2.0 Source: Reliability - Life Test Analysis Using the Weibull Distribution, N. Kaufman, M. Lipow; TRW Space Technology Laboratories, Redondo Beach, California (Presented at the 11th Western Regional Meeting of ASQC).

3.0 Purpose: Provide a system of selecting a lower confidence bound on $R(X) = \exp(-(X-1)^{\beta}/\alpha)$ or equivalently an upper bound on $F(X) = 1 - R(X)$ that are better in general than distribution free bounds. The first and last order statistic from a sample of size n are used.

4.0 Limitations of Method

4.1 The efficiency of the lower bound is unknown, i.e., the difference between the bound supplied by this method and the largest value possible for the bound using the same data is unknown.

4.2 Data on order statistics other than the first and last cannot be used.

4.3 Method does not work with censored data.

5.0 Criteria Used in Measuring Efficiency of Method

Characteristics of the confidence bound and the ease of computation are used.

6.0 Knowledge Required to Use the Method

6.1 Values of the first and last order statistic X_1 and X_n.

6.2 The value of X, X_o, of interest.

6.3 Desired confidence, α.

6.4 Sample size n.

6.5 The number of samples with values less than X_o, f.

7.0 Instructions for Use

7.1 Determine if X_1 is greater than, equal to or less than X_o.

7.2 If $X_1 > X_o$

7.2.1 Compute C_o from $C_o = 1-(1-\alpha)^{1/n}$

7.2.2 Compute C_w by solving

$$\ln \ln \frac{1}{1-C_w} = \ln\left(\frac{\ln\left(\frac{1}{p_1}\right)}{\ln\left(\frac{1}{1-p_1}\right)}\right)\frac{\ln(X_o/X_1)}{\ln(X_n/X_1)} + \ln \ln \frac{1}{1-p_1}$$

where p_1 is obtained from table A in the source or solving

$$2(1-p_1)^n - (1-2p_1)^n = 1 - \alpha$$

with n given.

7.2.3 Select the minimum of (C_o, C_w).

7.2.4 The lower confidence bound for $R(X_o)$ with $X_1 > X_o$ is then

$$1 - \text{Min}\ (C_o, C_w)$$

7.3 If $X_o \leq X_1$

7.3.1 Compute C_f by solving $\sum_{j=1}^{f} \binom{n}{j} C_f^{\ j} (1-C_f)^{n-j}$

Extensive tables for finding C_f are given in STL Report No. 6120-0008-MU-000, "Tables of Binomial Upper Confidence Limits on Probability of Failure," July 1961.

7.3.2 The lower confidence level for $R(X_o)$ with $X_1 \leq X_o$ is Max $(0,(1-C_f))$

8.0 Advantages of Method

8.1 The confidence bound is more efficient than distribution free methods (in general) when $X_1 > X_o$.

8.2 The computation is straightforward. (Tables for computing C_w are available in the source for n = 5,7,10,20 and α = .5, .9, .95.)

9.0 Reported Experience with Method: Not applicable.

5.2.12

1.0 Title of Method: Estimation of the Parameters of a Normal Population When the Sample is Censored Several Times.

2.0 Source: "Progressively Censored Samples in Life Testing," A. C. Cohen, Jr., Technometrics, Volume 5, No. 3, August 1963.

3.0 Purpose: This maximum likelihood estimator of μ and σ from a population with life times x distributed by

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \quad \exp\left(-\ 1/2\ \left(\frac{x-\mu}{\sigma}\right)^2\right)$$

when portions of the sample are removed before they fail and before the test ends.

4.0 Limitations of Method

Iterative computation is needed to find the estimators.

5.0 Criteria Used in Measuring Efficiency of Method

Ability to obtain the estimate and ease of computation were considered.

6.0 Knowledge Required to Use the Method

6.1 Sample Size N.

6.2 Number removed, r_i, at each censoring time, T_i.

6.3 Number of censoring times, k.

6.4 Number failed n and times of failure x_1, x_2 ...x_n.

6.5 Value of μ or σ if known.

7.0 Instructions for Use

7.1 μ and σ unknown.

7.1.1 Compute $\bar{x}$ from $\bar{x} = \sum_{i=1}^{n} x_i$

7.1.2 Compute s^2 from $s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

7.1.3 Solve iteratively

$$\bar{x} = \hat{\mu} - \hat{\sigma}\sum_{i=1}^{k} r_i Z_i/n$$

$$s^2 = \hat{\sigma}^2\left[1 - \sum_{i=1}^{k} r_i \xi_i Z_i/n - \left(\sum_{i=1}^{k} r_i Z_i/n\right)^2\right]$$

where

$$\xi_i = \frac{T_i - \mu}{\sigma} \text{ and } Z_i = \frac{\frac{1}{\sqrt{2\pi}}\exp(-\xi_i^2/2)}{1 - \int_{-\infty}^{\xi_i}\frac{1}{\sqrt{2\pi}}\exp(-t^2/2)\,dt}$$

7.1.4 Newton's Method of iteration or a method given in the source document are declared satisfactory for use in 7.1.3

8.0 Advantages of Method

It allows the use of all data from a progressively sampled life test.

9.0 Reported Experience with Method.

Examples are given in the source document.

5.2.13

1.0 Title of Method: Selection of Estimators for the Probability of Zero Failures, P_o, in m Binomial Trials when P_o is greater than 0.5.

2.0 Source: Estimation of the Probability of Zero Failures in m Binomial Trials, H. C. Rutemiller, Journal of the American Statistical Association, Volume 62, March 1967.

3.0 Purpose

This method indicates which estimator of P_o should be used when $P_o > .5$. The estimators considered are

3.1 Maximum likelihood (ML) $\hat{P} = (1 - \frac{r}{n})^m$ where r is the number of failures observed in n tests and m is the number of interest

3.2 Minimum Variance Unbiased $\tilde{P} = \binom{n-m}{r} / \binom{n}{r}$ $\quad r \leq n-m$

$= 0 \quad r > n-m$

where $\binom{y}{x}$ is the binomial coefficient $\frac{y!}{x!(y-x)!}$

4.0 Limitations of Method

The conclusions are based on observation of a limited number of cases.

5.0 Criteria Used in Measuring Efficiency of Method

The mean squared errors of the estimators were used.

6.0 Knowledge Required to Use Method

6.1 Knowledge that $P_o > .5$

6.2 n and r from a test.

7.0 Instructions for Use

7.1 If an unbiased estimator is desired, use $\tilde{P}_o$.

7.2 If bias is not important, use $\hat{P}_o$ since it has smaller mean squared error $E(\hat{P}_o - P_o)^2$.

8.0 Advantages of Method

8.1 The method supplies the best estimator in the sense of 7.1 and 7.2.

8.2 P_0 is an important statistic for one shot destructive devices such as detonation caps.

9.0 Reported Experience with Method: Not applicable.

5.2.14

1.0 Title of Method (Author's Title): Estimating the Parameters of Negative Exponential Populations from One or Two Order Statistics, H. L. Harder.

2.0: Source: Annals of Mathematical Statistics, Volume 32, 1961, pp. 1078-1090.

3.0: Purpose: This method gives the minimum variance unbaised estimates of the parameters σ and α from the negative exponential distribution.

$$f(x) = \frac{1}{\sigma} \exp\left(-(x-\alpha)/\sigma\right) \qquad \alpha \leq x \leq \infty$$

$$= 0 \qquad x < \alpha$$

based on one or two order statistics (x_k or x_ℓ, x_m).

4.0 Limitations of Method

4.1 Maximum likelihood estimators that are more efficient (smaller variance) can be computed if the first $m(m<n)$ order statistics are available.

4.2 Tables are given only for the best one or two order statistics for each n, hence the method requires extensive computation to use any other order statistics.

5.0 Criteria Used in Measuring Efficiency of Method

Efficiency of the estimators, ease of computation and ease of data collection were considered.

6.0 Knowledge Required to Use Method

6.1 Sample size n.

6.2 Value of the appropriate order statistics.

7.0 Instructions for Use

7.1 For $\alpha = 0$ known.

7.1.1 For the two order statistic estimate

7.1.1.1 Find ℓ and m for correct n from table I of the source document.

7.1.1.2 Observe x_ℓ and x_m

7.1.1.3 Compute $\tilde{\sigma}_{\ell m}$ the 2 order statistic estimator of σ from $\tilde{\sigma}_{\ell m} = C_\ell x_\ell + C_m x_m$

where C_ℓ and C_m are obtained from table I.

7.1.1.4 For ℓ and m ($\ell < m$) other than those tabled C_ℓ and C_m can be computed from:

$$C_\ell = 1\Big/\left(\sum_{i=1}^{\ell} a_i + \lambda \sum_{i=1}^{m} a_i\right)$$

$$C_m = \lambda C_\ell$$

where $\lambda = \sum_{i=\ell+1}^{m} a_i \sum_{i=1}^{\ell} a_i^2 \Big/ \left(\sum_{i=1}^{\ell} a_i \sum_{i=1}^{m} a_i^2 - \sum_{i=1}^{m} a_i \sum_{i=1}^{\ell} a_i^2\right)$

and $a_i = 1/(n-i+1)$

7.1.2 For the one order statistic estimator

7.1.2.1 Find k from table I.

7.1.2.2 Observe the value of x_k.

7.1.2.3 Compute $\tilde{\sigma}_k$ from $\tilde{\sigma}_k = C_k x_k$

where C_k is found in table I

7.1.2.4 For an order statistic other than X_k.

C_k is given by

$$C_k = 1\Big/\sum_{i=1}^{k} a_i$$

7.2 When α is unknown x_1 (the first order statistic) must be used as one of the two order statistics.

7.2.1 Find m from table II in the source document.

7.2.2 Observe the values of x_1 and x_m.

7.2.3 Compute $\tilde{\sigma}$, $\tilde{\alpha}$, and $\tilde{\mu}$ (μ is the mean of f(x); $\mu = \alpha + \sigma$)from

$$\tilde{\sigma} = C_\sigma\,(x_m - x_1)$$

$$\tilde{\alpha} = (1 + C_\alpha)x_1 - C_\alpha x_m$$

$$\tilde{\mu} = (1 + C_\alpha - C_\sigma)x_1 + (C_\sigma - C_\alpha)x_m$$

where C_α and C_σ are obtained from table II.

7.2.4 For other m's not given in table II C_α and C_σ are

$$C_\alpha = a_1 \Big/ \sum_{i=2}^{m} a_i$$

$$C_\sigma = 1 \Big/ \sum_{i=2}^{m} a_i$$

8.0 Advantages of Method

8.1 For the optimum order statistics computation is easy.

8.2 An estimate can be made from a limited amount of order statistics (any one if $\alpha = 0$ and x_1 and any other one if α is unknown)

9.0 Reported Experience with Method: Not applicable.

5.2.15

1.0 Title of Method: The Best Estimators of Reliability When the Failure Rate is Constant.

2.0 Source: The Efficiencies in Small Samples of the Maximum Likelihood and Best Unbiased Estimators of Reliability Function, S. Zachs, M. Even, Journal of the American Statistical Association, Volume 61, December 1966.

3.0 Purpose

3.1 This method indicates which estimator of reliability, $R(\tau)$, of an item having a constant failure rate λ is best when data is collected by (i) observing the number of failures X_i in time t_o of the i^{th} sample of size n. (If a part is not repairable, it is assumed that a new unit replaces the failed one and operates until t_o) and (ii) observing the times of failures $\{X_i\}$ of the n samples. (Note the X_i represent different variables in i and ii)

3.2 The estimators in case (i) are:

3.2.1 Minimum Variance Unbiased (MVU) $\hat{R}$; $\hat{R} = (1 - \zeta)^S$

where $\zeta = \tau/nt_o$ and $S = \sum_{i=1}^{n} X_i$

3.2.2 Maximum Likelihood (ML) $R = \exp(-\zeta S)$

3.3 The estimators in case (ii)

3.3.1 MVU $\hat{R}_n = (1 - \frac{\tau}{T})^{n-1}$

where $T = \sum_{i=1} X_i$

3.3.2 ML $\tilde{R} = \exp(-n\tau/T)$

4.0 Limitations of Method

4.1 Only two cases n = 4 and n = 8 were studied.

4.2 The results for different criteria are contradictory.

4.3 A knowledge of the value of $\lambda\tau$ is needed.

5.0 Criteria Used in Measuring Efficiency of Method

Efficiency relative to the Cauchy Rao lower bound and closeness function defined in the source.

6.0 Knowledge Required to Use Method

6.1 Type of data collection (i.e., (i) or (ii)).

6.2 Approximate range of product ($\lambda\tau$) where λ is the failure rate of the item and τ is time it must operate.

7.0 Instructions for Use

7.1 For data collection type i use

$\tilde{R}$ if $\lambda\tau < 1$

$\hat{R}$ if $\lambda\tau \geq 1$

from 3.2.

7.2 For data collection type (ii) use

$\hat{R}$ if $\lambda\tau \leq .2$

$\tilde{R}$ if $\lambda\tau > .2$

from 3.3.

8.0 Advantages of Method

It allows the choice of the best estimator of Reliability.

9.0 Reported Experience with Method: Not applicable.

5.2.16

1.0 Title of Method (Author's Title): Estimation of (Maximum Likelihood) Parameters of the Gamma Distribution Using Order Statistics, M. B. Wilks, R. Gnanadesikan and J. J. Huyett.

2.0 Source: Biometrika (1962) 49, 3 and 4, p. 525.

3.0 Purpose: Maximum likelihood estimation of the parameter λ, n, and α of a random variable having a gamma distribution

$$g(y;\lambda,n,\alpha) = \frac{\lambda^n}{\Gamma(n)} (y-\alpha)^{n-1} \exp(-\lambda(y-\alpha)) \quad y > \alpha$$

$$= 0 \quad y < \alpha$$

using the first M order statistics obtained from a sample of size K.

4.0 Limitations of Method

4.1 The estimators may be biased. Maximum likelihood estimators are generally asymptotically efficient and consistent (and sufficient if sufficient estimators exist) but they are not generally unbiased.

4.2 Interpolation will be necessary in most cases. Errors resulting from interpolation of the tables are possible.

4.3 The case of α unknown requires many iterative computations.

5.0 Criteria Used in Measuring Efficiency of Method

The characteristics of the estimators, the data collection problems and ease of computation were considered.

6.0 Knowledge Required to Use Method

6.1 The values of the Order Statistics $x_1', x_2', \ldots x_M'$

6.2 The value of α if known.

7.0 Instructions for Use

7.1 α known or assumed zero.

7.1.1 Subtract α from each order statistic: set $x_i = x_i' - \alpha$

7.1.2 Compute P and S from

$$P = \frac{\left(\prod_{i=1}^{M} x_i\right)^{1/M}}{x_M} \quad ; \quad S = \frac{\sum_{i=1}^{M} x_i}{M x_M}$$

7.1.3 Obtain η and μ from the appropriate K/M table in Appendix B of the source document. Note that the interpolation instructions in the front of Appendix B will probably be needed.

7.1.4 Compute $\hat{\lambda}$ from $\hat{\lambda} = \dfrac{\hat{\eta}}{\hat{\mu} x_M}$

7.2 α unknown.

7.2.1 Select a trial value of α and perform 7.1.1 through 7.1.3 to obtain values of η and μ.

7.2.2 Compute $\log \mathcal{L}$ from

$$\log \mathcal{L} = K\eta \log(\eta/\mu) - K \log \Gamma(\eta) - M \log(x_M - \alpha)$$
$$+ (\eta - 1) \log P - M(\eta/\mu) S$$
$$+ (K-M) \log\left[\int_1^\infty t^{\eta-1} \exp(-(\eta/\mu)t)dt\right]$$

7.2.3 Repeat 7.2.1 and 7.2.2 until a maximum value of $\log \mathcal{L}$ appears. Use the last value of α as $\hat{\alpha}$ and repeat 7.1.1 through 7.1.4 with $\hat{\alpha}$ to find $\hat{\eta}$ and $\hat{\lambda}$.

8.0 Advantages of Method

8.1 It produces maximum likelihood estimators.

8.2 It can use censored ($M < K$) data.

9.0 Reported Experience with Method

9.1 An example with real data assuming $\alpha = 0$ was given in the source.

9.2 The source reports on a test case for α unknown. Data from a gamma distribution with $\alpha = 4$, $\lambda = 1/2$, $\eta = 2$, $K = 30$ and $M = 25$ were drawn. The resulting estimates were $\hat{\alpha} = 4.38$, $\hat{\eta} = 0.350$ and $\hat{\lambda} = 1.499$.

5.2.17

1.0 Title of Method (Author's Title): Scale Parameter Estimation of the Gamma Probability Function Based on One Order Statistic, R. C. Karns.

2.0 Source: Defense Documentation Center Cameron Station, Alexandria, Virginia, AD 425 223.

3.0 Purpose: This method makes point and interval estimates of the scale parameter θ from the Gamma Distribution

$$f(x) = \frac{1}{\theta^{\alpha}\Gamma(\alpha)} x^{\alpha} \exp(-x/\theta)$$

with parameters α and θ when α is known based on one order statistic.

($\Gamma(\alpha)$ is the Gamma function $\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1}\exp(-x)dx$.)

4.0 Limitations of Method

4.1 The value of the shape parameter α must be known.

4.2 The estimates are not very efficient compared to maximum likelihood estimates which are easy to compute.

4.3 The method can only be used with the best order statistic since this is the only one tabled.

4.4 The best order statistic (the most efficient) is generally large compared to the sample size so that not much time is gained by this method.

5.0 Criteria Used in Measuring Efficiency of Method

The characteristics of the estimates, ease of data collection and ease of computation were considered.

6.0 Knowledge Required to Use Method

6.1 The value of α.

6.2 The sample size n.

6.3 The confidence desired 1-P.

6.4 The value of the appropriate order statistic m, $t_{n,m}$.

7.0 Instructions for Use

7.1 To obtain a point estimate $\bar{\theta}$.

7.1.1 Look in table I of the source at the values of α and n (A and N in the table) and determine m (M) and the multiplying factor (θ/T_M).

7.1.2 Compute $\bar{\theta}$ from $\bar{\theta} = (\theta/T_M)t_{n,m}$

7.2 To obtain interval estimates.

7.2.1 To obtain an upper bound look in table II in the source for α, n and (1-P) (in column 4) and read m (column 3) and the coefficient in column 5 (Up B/T(m))

7.2.2 Compute B_u (1-P) from

$$B_u\ (1-P) = (\text{Up } B/T(M))\ t_{n,m}$$

7.2.3 To obtain a two sided interval estimate look in column 9, table II in the source for the values of α, n, and (1-2P) (in column 7). If an asterisk appears, look in table III for the same α, n, and 1-2P.

7.2.4 From either table II or III, read m (column 3) and the coefficients in columns 5 and 6.

7.2.5 Compute the confidence bounds B_u (1-2P), B_L (1-2P) from

$$B_u\ (1-2P) = (\text{Up } B/T(M))\ t_{n,m}$$

$$B_L\ (1-2P) = (\text{Low } B/T(M))\ t_{n,m}$$

7.3 Tables I, II, and III give measures of efficiency of the estimates.

8.0 Advantages of Method

8.1 The computation is easy.

8.2 $\bar{\theta}$ is unbiased.

8.3 Data recording is minimal.

9.0 Reported Experience with the Method: Not applicable.

5.3 Distribution Free Methods.

5.3.1

1.0 Title of Method: A Distribution Free Test for Increasing Failure Rate.

2.0 Source: Tests for Monotone Failure Rate, F. Proschan and R. Pyle, Fifth Berkeley Symposium on Mathematical Statistics and Probability, July 1965 and Mathematical Note No. 400, Boeing Scientific Research Laboratories, March 1965.

3.0 Purpose: To test if a population has an increasing failure rate based on a set of times to failure.

4.0 Limitations of Method

4.1 In cases where the scale parameter λ of Weibull distribution $F_W(x) = 1 - \exp(-\lambda x^{\theta})$ or of the Gamma distribution

$f_G(x) = \dfrac{\lambda^{\theta} x^{\theta-1} \exp(-\lambda x)}{\Gamma(\theta)}$ are large, it is better to assume that the population has either the Gamma as the Weibull distribution instead of using this method.

4.1.1 The asymptotic relative efficiency, ARE, of the likelihood ratio test of an exponential against the Gamma when the distribution is actually Weibull, ARE_{GW} increases with λ and exceeds the distribution free test ARE which decreases with λ.

4.1.2 4.1.1 is true with Gamma and Weibull interchanged, i.e., a test against the Weibull when the distribution is actually Gamma has the same property for large λ.

4.2 For samples of more than 10 extensive computations are required to find the critical value for the test, $v_{n\alpha}$.

4.3 For large samples, a large number of computations are necessary. There are $n(n-1)/2$ comparison for a sample of size n.

5.0 Criteria Used in Measuring Efficiency of Method

The assumption required and the ease of computation were considered.

6.0 Knowledge Required to Use the Method

6.1 The values of the test results in order of Magnitude $(T_{n1}, T_{n2} \ldots T_{nn})$ from a sample of size n are required.

6.2 The level of significance $1-\alpha$.

7.0 Instructions for Use

7.1 Compute D^*_{ni} from $D^*_{n1} = T_{n1}$ and

$$D^*_{ni} = T_{ni} - T_{n(i-1)} \quad i = 2, 3 \ldots n$$

7.2 Compute D_{ni} from $D_{ni} = (n-i+1)\, D^*_{ni}$

7.3 Compute V_{ij} from

$$V_{ij} = \begin{cases} 1 & D_{ni} \geq D_{nj} \quad i, j=1,2, \ldots,n \\ 0 & \text{otherwise} \end{cases}$$

7.4 Compute V_n from $V_n = \sum_{i=1}^{n} \sum_{j=i+1}^{n} V_{ij}$

7.5 Determine $v_{n\alpha}$ from 7.5.1 or 7.5.2

7.5.1 For $n \leq 10$ from tables in either: A New Measure of Rank Correlation, M. G. Kendall, Biometrica, 30, pp. 81-93 or Nonparametric Tests Against Trend, H. B. Mann, Econometrica, 13, pp. 245-249. Look in the tables for the first value of k, say k_o such that the

$P_r(V_n \leq k) > 1 - \alpha$. Set $v_{n\alpha} = k_o$

7.5.2 For $n > 10$

7.5.2.1 Compute $P_n(k)$ for $k = 0, 1, \ldots n$ from

$$P_n(k) = P_{n-1}(k) + P_{n-1}(k-1) + \ldots P_{n-1}(k-n+1)$$

with $P_n(k) = 0$ for $k < 0$. The initial values can be obtained from the tables referenced in 7.5.1 by subtracting succeeding values of $P_r(V_n \leq k)$ and multiplying by n!

7.5.2.2 Compute the $P_r(V_n \leq k)$ from

$$P_r(V_n \leq k) = \frac{1}{n!} \sum_{j=1}^{k} P_n(j) \quad \text{for } k = 0, 1, 2 \ldots, n.$$

7.5.2.3 Find the first value of $k(k_o)$ such that $P_r(V_n \leq k) > 1 - \alpha$ and set $v_{n\alpha} = k_o$.

7.6 Reject the null hypothesis that the failure rate is constant if $V_n > v_{n\alpha}$. Thus, accepting the alternative hypothesis that the distribution has increasing failure rate.

8.0 Advantages of Method

8.1 It does not require an assumption about the form of the distribution.

8.2 The knowledge that the failure rate is increasing is valuable, see Method 5.3.2.

8.3 For λ close to 1, this method is superior (lower ARE) to picking the wrong distribution, i.e., testing against Gamma (Weibull) when the distribution is actually Weibull (Gamma).

8.4 For $n \leq 10$ computation is easy.

9.0 Reported Experience with Method: Not applicable.

5.3.2

1.0 Title of Method (Author's Title): Nonparametric Life Test Sampling Plans, R. E. Barlow, Shanti S. Gupta, University of California, Berkeley, California, July 30, 1964.

2.0 Source: Defense Documentation Center, AD-613 273

3.0 Purpose: Determine the sample size n and acceptance number c for a fixed time life test of the mean or quantiles of distribution that are characterized by only the following information:

i) The range of the distribution is 0 to ∞.

ii) The failure rate is increasing (or decreasing) with time.

4.0 Limitations of Method

4.1 It is necessary to know if the failure rate is increasing or decreasing.

4.2 The tests require more samples than those having more assumption about the distribution. If more information is available, distribution dependent tests should be used.

4.3 The sample sizes given consider only the consumer's risk. No provision is made for including the producer's risk.

5.0 Criteria Used in Measuring Efficiency of Method

The sample size and the ability to obtain test risks with limited assumptions were considered.

6.0 Knowledge Required to Use the Method

6.1 Parameter (mean or quantile) of interest.

6.2 Whether the failure rate is increasing or decreasing.

6.3 Number of failures, c.

6.4 Consumer's risk, 1-P*.

7.0 Instructions for Use

This method finds the sample size n required to make the following tests. The tests are passed if the number of failures is less than c + 1.

7.1 If the failure rate is increasing.

7.1.1 If the mean μ° is less than the prescribed value $\mu^{\circ *}$, the probability of passing the test is less than or equal to 1-P*.

7.1.1.1 Compute λ from $\lambda = t/\mu^{\circ *}$.

7.1.1.2 Enter table I of the source document at P*, c, and λ and observe n.

7.1.2 If the q^{th} quantile ζ_q is less than the prescribed value ζ_q^* the probability of passing the test is less than 1-P*.

7.1.2.1 Compute λ from $\lambda = t/\zeta_q^*$.

7.1.2.2 Enter table II at P*, q, c, and λ and observe n.

7.2 If the failure rate is decreasing

7.2.1 If $\zeta_q < \zeta_q^*$ than the probability of passing the test is less than or equal to 1-P*.

7.2.2 Compute λ from $\lambda = t/\zeta_q^*$

7.2.3 Enter table III at P*, q, c and λ and observe n.

8.0 Advantages of Method

It provides plans (n and c) that give known probabilities of consumer's risk that are distribution free.

9.0 Reported Experience with Method: Not applicable.

5.3.3

1.0 Title of Method: Distribution Free Estimators and Confidence Limits for Reliability During Development.

2.0 Source: Maximum Likelihood Estimation and Conservative Confidence Interval Procedures in Reliability Growth and Debugging Problems, R. E Barlow, F. Proschan and E. M. Scheuer, Memorandum RM 4749, NASA, January 1966, published by the Rand Corporation, Santa Monica, California.

3.0 Purpose: This method permits estimation of the reliability at the current stage of development of a device that has improving reliability during development. The only assumption about the distribution of life times in any particular stage is that the failure rate is constant or increasing. The fact that the failure rate is increasing at any stage does not mean that the reliability cannot improve in the next stage. It happens by obtaining a failure rate function (still increasing) which is everywhere less than it was at the previous stage. Conservative confidence bounds are also provided. A conservative confidence bound is one where the assurance is at least instead of exactly equal to a specified value that the reliability falls in a designated confidence set determined from the observations.

4.0 Limitations of Method

4.1 The confidence bounds are quite large.

4.2 The estimators are not generally unbiased.

4.3 The efficiency of the estimator is not given in the source.

4.4 Data must be retained for long periods, e.g., the length of time from the first stage of development to the last may be several years.

4.5 The table for $\epsilon_{n,\alpha}$ is restricted to n = 5, 8, 10, 20, 40, 50 and α = .1, .05, .01, .001 for small n.

5.0 Criteria Used in Measuring Efficiency of Method

The minimization of assumptions regarding the life distributions for the different development stages, ease of computation, and properties of the estimators were considered.

6.0 Knowledge Required to Use Method

6.1 Knowledge that reliability is improving from stage to stage.

6.2 Knowledge that failure rate is nondecreasing in any stage.

6.3 Observations of (1) successes and failures in each stage, or (2) lifetimes of samples in each stage.

7.0 Instructions for Use

7.1 If success and failures are observed, i.e., the number of successes x_i out of n_i trials in the i^{th} stage.

7.1.1 To compute the estimate of reliability $\hat{R}$.

7.1.1.1 Compute the $\{p_i\}$ for i=1, 2, ...k from

$$p_i = x_i/n_i$$

7.1.1.2 If $p_1 \leq p_2 \leq p_3 \ldots \leq p_k$ than the p_i are the estimators for each stage of development, i.e., $\hat{R}_i = p_i$

7.1.1.3 If for any j $p_j > p_{j+1}$ combine the observations on the j^{th} and (j+1)st stages and set

$$p_j = p_{j+1} = \frac{x_j + x_{(j+1)}}{n_j + n_{(j+1)}}$$

7.1.1.4 Repeat 7.1.1.2 and 7.1.1.3 until 7.1.1.2 leads to the estimators.

7.1.2 To compute the conservative 100 (1-α) percent lower confidence bound on R_K.

7.1.2.1 Compute x from $x = \sum_{i=1}^{k} x_i$

7.1.2.2 Compute n from $n = \sum_{i=1}^{k} n_i$

7.1.2.3 Use the standard approach for the lower confidence bound on the binomial parameter given in: Introduction to the Theory of Statistics, A. M. Mood, F. A. Graybill, 2nd ed., McGraw Hill Book Company, New York

7.2 If lifetimes $\{x_{ij}\}$ (i=1, 2, ..., k; j=1, 2, ..., n_i) are observed during the k stages of development.

7.2.1 To compute an estimate of reliability at time $t, \hat{R}(t)$.

7.2.1.1 Compute $\{y_{ij}(t)\}$ from $y_{ij}(t) = \begin{cases} 1 & \text{if } x_{ij} > t \\ 0 & \text{otherwise} \end{cases}$

7.2.1.2 Compute $\overline{F}_{in_i}(t)$ from $\overline{F}_{in_i}(t) = \frac{1}{n_i} \sum_{j=1}^{n_i} y_{ij}(t)$

7.2.1.3 If $\overline{F}_{1n_1}(t) \le \overline{F}_{2n_2}(t) \le \overline{F}_{3n_3}(t) \le \ldots \le \overline{F}_{kn_k}(t)$

then set $\hat{R}_i(t) = \overline{F}_{in_i}(t)$

7.2.1.4 If for some j $\overline{F}_{jn_j}(t) > \overline{F}_{(j+1)n_{j+1}}(t)$ combine

the data from the j^{th} and $(j+1)$st stage and perform 7.2.1.1 and 7.2.1.2 for the combined j^{th} and $(j+1)$st stage.

7.2.1.5 Repeat 7.2.1.3 and 7.2.1.4 until 7.2.1.3 leads to the estimators.

7.2.2 To obtain a conservative 100 $(1-\alpha)$ percent upper confidence curve on $F_k(t)$ for all $t > 0$, where $F_k(t)$ is the distribution function of the time to failure τ, i.e., $F_k(t) = P_r(\tau \le t)$.

7.2.2.1 Order all the x_{ij} from smallest to largest obtaining $x_1^\circ, x_2^\circ, \ldots x_n^\circ$

where $x_1^\circ \le x_2^\circ \le \ldots x_n^\circ$ and $n = \sum_{i=1}^{k} n_i$

7.2.2.2 Compute $\hat{F}(t)$ from $\hat{F}(t) = \frac{i(t)}{n}$

where $i(t)$ is the largest integer such that $x_i^\circ < t$.

7.2.2.3 Obtain the upper confidence curve on $F_K(t)$ by adding $\epsilon_{n\alpha}$ to each value of $\hat{F}(t)$. Obtain $\epsilon_{n\alpha}$ from a table on page 28 of the source document.

8.0 Advantages of Method

8.1 Estimates and confidence bounds are obtainable with limited assumptions on the distribution of time to failure.

8.2 The computation is not difficult.

8.3 The estimators are maximum likelihood estimators although the authors point out that "they do not necessarily enjoy all the desirable properties of maximum likelihood estimators in other situations."

9.0 Reported Experience with Method

Numerical examples are given in the source documents.

5.3.4

1.0 Title of Method: Distribution Free Estimation and Confidence Bounds for the Failure Rate from Data Gathered during Debugging.

2.0 Source: Maximum Likelihood Estimation and Conservative Confidence Interval Procedures in Reliability Growth and Debugging Problems, R. E. Barlow, F. Proschan and E. M. Scheuer, Memorandum RM 4749, NASA, January 1966, published by the Rand Corporation, Santa Monica, California.

3.0 Purpose: This method supplies the estimate of failure rate at time t, $r(t)$, during debugging when n failures have occurred during the debugging period (0 to t_o). Conservative confidence bounds[1] are also obtained.

4.0 Limitations of Method

4.1 The quantitative properties (bias, efficiency, etc.) of the estimators are unknown.

4.2 The confidence bound is likely to be large.

5.0 Criteria Used in Measuring Efficiency of Method

The properties of the estimators, width of the confidence bound, ease of computation and ease of data collection were considered.

6.0 Knowledge Required to Use Method

6.1 Knowledge that the failure rate $r(t)$ is decreasing with time.

6.2 Observation of times between failure $\{x_{ij}\}$ where j identifies the system $j = 1, 2, \ldots k$ and i represent the failure number for the j^{th} system $i = 1, 2 \ldots, n_j$.

7.0 Instructions for Use

7.1 To compute the estimator, $\hat{r}(t)$, of $r(t)$.

7.1.1 Compute the times of failure $\{S_{ij}\}$ from $S_{ij} = \sum_{v=1}^{i} x_{vj}$

7.1.1.1 Order all the S_{ij} from smallest to largest to obtain $T_1, T_2, \ldots T_n$ where $n = \sum_{j=1}^{k} n_j$ and T_k is the k^{th} largest S_{ij}.

[1] See 5.3.3 for the definition of conservative confidence bounds.

7.1.2 Compute X_i from $X_i = \left\{N_i\left[T_i - T_{(i-1)}\right]\right\}$ where N_i is the number of systems operating in the internal $(T_{(i-1)}, T_i)$ and $T_0 = 0$.

7.1.3 If $X_1 \leq X_2 \leq X_3 \ldots \leq X_n$ then set $\hat{r}(t)$ equal to

$$\hat{r}(t) = \frac{1}{X_{i(t)}}$$

where $i(t)$ is the largest integer such that $\sum_{i=1}^{i(t)} N_i T_i \leq t$

7.1.4 If for some j, $X_j > X_{j+1}$, replace X_j and X_{j+1} by X_j^* and X_{j+1}^* where $X_j^* = X_{j+1}^* = \left[\frac{1}{2}(X_j + X_{j+1})\right]$

7.1.5 If more than two consecutive X_i's must be used to eliminate reversals in 7.1.3, say j, $j+1$, $j+2$, ..., $j+m$ then set $X_j^* = X_{j+1}^* = \ldots X_{j+m}^* = \left[\frac{1}{m}\sum_{v=1}^{m} X(j+v)\right]$

7.1.6 Repeat 7.1.3 using X_i^*'s where necessary to find the $\hat{r}(t)$.

7.2 To obtain the 100 $(1-\alpha)$ percent conservative upper confidence bound on the failure rate at the end of debugging r_o (r_o is assumed independent of t) pool all the data and compute the upper bound for an exponential distribution.

7.2.1 Compute the upper bound $\bar{r}_o$ from

$$\bar{r}_{o,(1-\alpha)} = \chi^2_{1-\alpha}(2n) \Big/ 2\sum_{j=1}^{k}\sum_{i=1}^{n_j} X_{ij}$$

where $\chi^2_{1-\alpha}(2n)$ is the value of the Chi square distribution with $2n$ degrees of freedom that is exceeded 100α percent of the time.

8.0 Advantages of Method

8.1 Estimates can be made without assumptions about the distribution.

8.2 The estimates are maximum likelihood estimates although the authors say that they may not have all the desirable properties of maximum likelihood estimators.

8.3 The computation of the estimators and the confidence bound is relatively easy.

9.0 Reported Experience with Method

Numerical examples are given in the source document.

6.0 THE MULTIPLE MODES OF FAILURE PROBLEM

6.1 Introduction

In accelerated life tests (ALT's) it is generally felt that if the "accelerating" conditions result in changes in failure mode (from those observed at normal conditions), the accelerated test is not valid. It is the purpose of this section to investigate, statistically, the validity of this presumption. Now, a large number of electronic, electromechanical and mechanical parts exhibit (even though there may be a "dominant" mode of failure) more than one mode of failure. Although in the past, failure data has been well described by certain statistical models (e.g., the Weibull distribution) much failure data remains unexplained. For example, in order to explain some observed failure results composite Exponentials, composite Weibulls, etc., have been used. Another purpose of this section then is to see if the occurrence of more than one mode of failure and a statistical model describing the part failure behavior can explain some previously unsatisfactorily explained part failure data.

As pointed out (see, for example, Reference 350) two very appealing models exist to explain multiple modes of failure. They are

1) The competing risks model (CR).

2) The mixed population model (MP).

Both models apply not only to "basic components" but to "systems" composed of components. The basic unit to be treated here is a part, whether it be a single component or composed of components.

6.2 The Competing Risks Model

6.2.1 Description of CR model.

Suppose a part exhibits K modes of failure $M_1, M_2, ---, M_K$ and that a random lifetime on this part occurs as follows:

When part operation begins, each failure mode begins, (according to its own failure distribution, and simultaneously and independently from the other modes) a random lifetime. Thus K lifetimes begin. Let the length of these lifetimes be denoted by the set $(X_1, ---X_K)$. The subscript corresponds to the particular mode of failure. Now failure time, say X, is clearly

$$X = \text{Minimum } (X_1, ---, X_K) \qquad (6.2.1.1)$$

and the fact that failure may be due to mode i (and hence the other K-1 X's are not observable) does not prevent one from symbolically writing down the set $(X_1, ---, X_K)$.

The random life time X is called the first order statistic and has distribution function (d.f.)

$$F_X(x) = P(X \le x) = 1 - \prod_{i=1}^{K} \left[1-F_{X_i}(x)\right] \qquad (6.2.1.2)$$

6.2.2 The CR for the exponential case.

The CR model provides a convenient explanation of the oft observed result that part failure times are exponential even though several modes of failure are observed. Suppose, as always, there are K modes of failure, that the CR model holds and that the d.f. for each mode is exponential, i.e., suppose

$$F_{X_i}(x) = 1 - e^{-\lambda_i x}; x, \lambda_i > 0 \qquad (6.2.2.1)$$

Then the distribution of times to part failure is given (from 6.2.1.2) by

$$F_X(x) = 1 - \prod_{i=1}^{K} \left[e^{-\lambda_i x}\right] = 1-e^{(\sum^{K} \lambda_i)x} \qquad (6.2.2.2)$$

(6.2.2.2) is thus exponential with failure rate

$$\lambda = \sum^{K} \lambda_i \qquad (6.2.2.3)$$

and the CR model explains the observation of an exponential failure distribution in the face of K modes of failure. Equation (6.2.2.2) also illustrates why the decomposition of λ into its constituent parts (e.g., for prediction purposes) is perfectly acceptable in this case.

While reasonably rare, it may happen that it is desirable to estimate each λ_i from failure data (say, for part design improvement). Whether rare or not, an analysis the estimation problem sheds additional light on the multiple modes of failure problem. Suppose then, it is desired to estimate the λ_i from failure data. Suppose further, that n failures have occurred: n_i of mode i so that

$$\sum^{K} n_i = n \qquad (6.2.2.4)$$

and that $N \ge n$ parts were originally on test. It is easy to show that the maximum likelihood estimate (MLE) of λ_i given by $\hat{\lambda}_i = \frac{n_i}{T}$

where T = the total life of all N parts, i.e.,

$$T = \sum^{n} x_{\cdot,j} + (N-n)\left[x_{\cdot,n}\right]. \qquad (6.2.2.5)$$

where the "dot" refers the particular mode causing failure and $x_{\cdot,n}$ is the longest lifetime. The estimate of λ, i.e., $\hat{\lambda}$, is as expected

$$\hat{\lambda} = \frac{n}{T} = \frac{n_1 + \text{---} + n_K}{T} = \frac{\sum^{K} n_i}{T} = (\hat{\lambda}_1 + \text{---} + \hat{\lambda}_K). \qquad (6.2.2.6)$$

Another, more intuitive approach, to the estimation of the λ_i is obtained by asking the question: What is the relative frequency of occurrence of failures due to mode i? More specifically, what is the probability that a given part will fail due to mode i? For the exponential CR model the answer is

$$P(\text{failure due to mode i}) = \frac{\lambda_i}{\sum^{K} \lambda_i} \qquad (6.2.2.7)$$

To obtain the MLE estimate of the quantity $\frac{\lambda_i}{\sum^{K} \lambda_i}$

several approaches are possible. One is to consider each failure time as a Bernoulli trial with "success" being failure due to mode i and "failure" being failure due to any other mode. Then the MLE estimate of the P(mode i caused failure) is

$$\frac{n_i}{n} \qquad (6.2.2.8)$$

and since

$$\frac{n_i}{n} = \frac{n_i}{T}\bigg/\frac{n}{T} = \frac{\hat{\lambda}_i}{\hat{\lambda}} \text{ then } \hat{\lambda}_i = \frac{n_i}{T}, \text{ hence} \qquad (6.2.2.9)$$

another method of deriving the MLE of λ_i is available.

Now, suppose that the failure times under accelerated and normal conditions are both exponential with failure rates λ_A, λ_N respectively and that under accelerated conditions there are K_A modes of failure and under normal conditions there are K_N modes of failure ($K_A \gtrless K_N$). It is shown in various places (e.g., see Reference 297) that the time to failures under accelerated and normal conditions are related by a simple linear transformation and a transformation function permits derivation of an algorithm to

estimate λ_N from an estimate of λ_A. So far, the mathematics in no way depends for its validity on the preservation of failure modes. However, inspecting

$$\lambda_A = \lambda_{1,A} + \lambda_{2,A} + \cdots + \lambda_{K_A,A}$$

$$\lambda_N = \lambda_{1,N} + \lambda_{2,N} + \cdots + \lambda_{K_N,N} \qquad (6.2.2.10)$$

it may turn out: (1) there are modes of failure observed under normal conditions not observed under accelerated conditions and conversely. That is, if $\lambda_{i,A}$ represents the failure rate due to mode i under accelerated conditions there may be no corresponding $\lambda_{i,N}$ (2) that there is a corresponding mode i but that

$$\lambda_{i,A} > \lambda_{i,N} \quad \text{or} \quad \lambda_{i,N} > \lambda_{i,A}$$

or (3) a combination of (1) and (2). It is clear now that if the purpose of the accelerated test is to estimate λ_N from an estimate of λ_A without regard to estimating the individual λ_i comprising λ_N it is mathematically and practically immaterial whether the composition of λ_A and λ_N is different provided only that the magnitude of the relationship between λ_N and λ_A (as given by the transformation function) can be expected to persist. However, if the purpose of the (accelerated) test is to estimate some or all of the individual $\lambda_{i,N}$ then those $\lambda_{i,N}$ must be represented in λ_A although it is not particularly bothersome that $\lambda_{i,A} \neq \lambda_{i,N}$ provided only the relationships between $\lambda_{i,N}$ and $\lambda_{i,A}$ can be expected to persist.

6.2.3 The CR model for the Weibull case with equal β's.

Suppose again the CR model with

$$F_{X_I}(x) = 1 - e^{-\alpha_i x^{\beta}} \quad \alpha_i,\ \beta,\ x > 0 \qquad (6.2.3.1)$$

In (6.2.3.1) the scale parameter of the Weibull is commonly referred to as α_i^{-1} and it has been assumed γ (the guarantee time) is zero. Note that the d.f. for each individual failure mode has been assumed to be Weibull and that all i of the K modes have the same shape parameter β. This is certainly an unusual situation but it is worth looking into. From (6.2.1.2)

$$F(x) = 1 - e^{-\sum^{K} \alpha_i x^{\beta}} \qquad (6.2.3.2)$$

$$= 1 - e^{-x^{\beta} \sum^{K} \alpha_i}.$$

Hence, X has the Weibull distribution with shape parameter β and scale parameter $(\sum^{K} \alpha_i)^{-1}$. Now the MLE's are available for $(\sum^{K} \alpha_i)^{-1}$ and for the individual α_i^{-1}'s but as usual for the Weibull the MLE's are not available in closed form, i.e., iterative solution techniques must be used. However, an alternate approach is available. As in the exponential case:

$$P(\text{mode } i \text{ causes failure}) = \frac{\alpha_i}{\sum^{K} \alpha_i} \tag{6.2.3.3}$$

and treating each failure time as a Bernoulli trial with "success" if the i^{th} mode caused failure and "failure" if it did not (i.e., some other mode caused failure) then the MLE's of the left hand side of (6.2.3.3) are: $\frac{n_i}{n}$ and the K equations

$$\frac{n_i}{n} = \frac{\tilde{\alpha}_i}{\sum^{K} \tilde{\alpha}_i} \tag{6.2.3.4}$$

Along with $\sum \tilde{\alpha}_i = \tilde{\alpha}$, where $\tilde{\alpha}$ is estimated from say, Weibull probability paper, result in estimates for each α_i.

As in the exponential case, if the failure times at accelerated and normal conditions are Weibull (each mode having the same β but possibly differing α's) then a transformation function (see either Reference Reports 297 or 350) relating the two exists and it is immaterial whether the failure modes change or not, provided only that the transformation function persists from test to test. On the other hand, if it is desired to estimate the individual $\alpha_{i,N}$'s they must be present (even though it is not necessary that $\alpha_{i,N} = \alpha_{i,A}$) in α_A. In this case, transformation functions can still be achieved.

In summary, suppose one observes a "good" Weibull fit to some failure data exhibiting more than one failure mode. Then the CR model provides an acceptable explanation of what is happening.

Perhaps, it is wise to call attention to something that should be fairly obvious by now:

The individual failure mode distribution functions are unobservable unless some physical means of "tying off" the

other K-1 modes of failure is available. That is, each failure time (for the i^{th} mode) is a sample from

$F_{X_i}(x_i | x_i < x_j \text{ all } j \neq i)$ not from $F_{X_i}(x_i)$.

6.2.4 The CR model for the general Weibull case.

Suppose again the CR model and now that for each mode i

$$F_{X_i}(x) = 1 - e^{-\alpha_i x^{\beta_i}} \quad \alpha_i, \beta_i, x > 0 \quad i = 1, \ldots, K \quad (6.2.4.1)$$

i.e., each d.f. has differing α's and β's.

Then

$$F_X(x) = 1 - e^{-\sum^{K} \alpha_i x^{\beta_i}} \quad (6.2.4.2)$$

Note again the usual notation for the Weibull scale parameter is not α_i but α_i^{-1}; however, this notational convenience has nothing to do with the development.

Now on inspecting (6.2.4.2) it is clear that the times to failure (i.e., the failure data) will no longer exhibit a Weibull distribution (unless by chance) since $F_X(x)$ is not Weibull. Although no "name" exists for (6.2.4.2), it is clearly the distribution of the first order statistic based on a sample of size K each element from each of K different Weibulls.

The purpose of the following discussion is to make the following point:

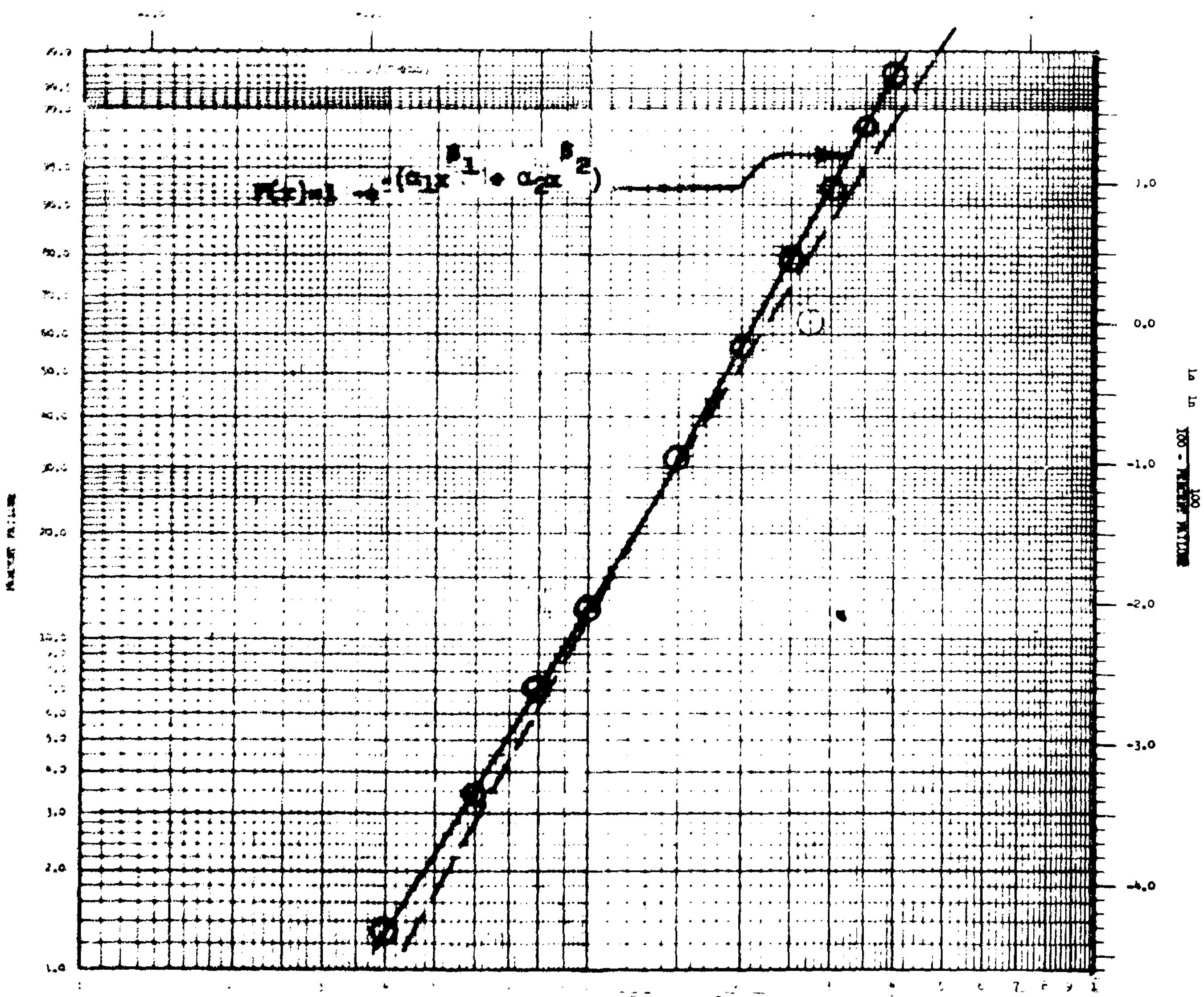
1) The CR model (general Weibull case) often looks very much like an ordinary Weibull and having shown 1) above, one can then say the CR model is quite consistent with data which has been observed to fit the Weibull.

Regarding 1) above, consider the following two (2) CR models for the general Weibull case where the number of failure modes is two. That is suppose:

i) $\alpha_1 = .05 \quad \beta_1 = 2$ ii) $\alpha_1 = .03 \quad \beta_1 = 1.5$

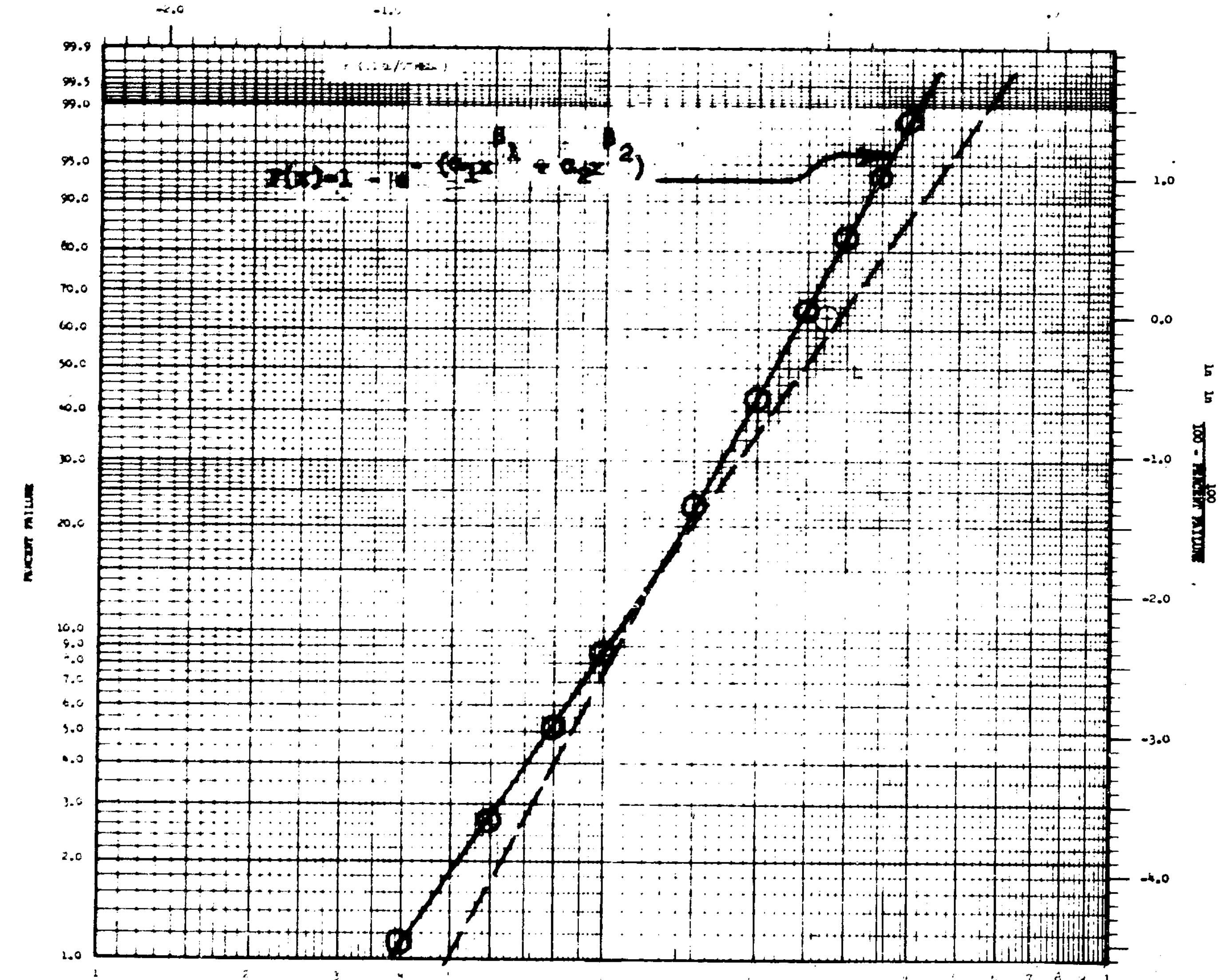
$\alpha_2 = .08 \quad \beta_2 = 3$ $\alpha_2 = .06 \quad \beta_2 = 3.0$

The plots of these two c.d.f.'s (on Weibull paper) are shown on the following pages(Figures 6.2.4.1 and 6.2.4.2). It is immediately seen from the figures that both graphs



$\alpha_1=.05,\ \alpha_2=.08,\ \beta_1=2,\ \beta_2=3$

Figure 6.2.4.1. CR WEIBULL PLOT(K=2)



$\alpha_1 = .03, \alpha_2 = .06, \beta_1 = 1.5, \beta_2 = 3.0$

Figure 6.2.4.2. CR WEIBULL PLOT (M=2)

take the approximate form of a straight line that looks "bent" at some point $(x_0, F_X(x_0))$. The two straight segments which make up each graph are shown, along with their extensions (in dotted lines) to emphasize the degree of bending. In general, it is observed that for each set $\{\alpha_1, \alpha_2, \beta_1 \beta_2\}$ that determines a CR model for Weibull failures in two modes, there exists a point $x_0 > 0$ and numbers $\alpha', \beta', \alpha'', \beta''$, such that the ordinary Weibull c.d.f. with parameters $\{\alpha', \beta'\}$ $(\{\alpha'', \beta''\})$ is a "good" approximation to the CR Weibull c.d.f. in the region $x \leq x_0$ $(x \geq x_0)$. Also, by inspecting (6.2.4.2), it is obvious that if $\alpha_1 \cong \alpha_2$ and/or $\beta_1 \cong \beta_2$, then the CR model looks like a single ordinary Weibull over the whole range of x. Even with rather large differences in the α_i and β_i, the approximating parameters α', α'' (β', β'') will be nearly equal, because of the damping effect of the logarithmic transformation. In fact, in most cases of interest, the CR Weibull (K=2) closely resembles an ordinary Weibull. In actual practice, to distinguish a CR Weibull (K=2) - even one with a large degree of "bend" - from the ordinary Weibull would require taking a very large (and hence impractical) number of observations.

6.2.5 The CR model for the general case.

Even when the families of the failure distribution are the same for all K modes, e.g., all Weibull, the problems of estimating the parameters are somewhat difficult. When, in the general case, the distributions $F_{X_i}(x)$ are different with respect to parameters and family, very little, if anything, has been done regarding estimation. Obviously, $F_X(x)$ (see 6.2.1.2) has little chance of being available in closed form.

6.3 The Mixed Population Model

6.3.1 Description of the MP model.

As an explanation of the multiple modes of failure problem, the MP model is somewhat less satisfying than the CR model of Section 6.2. This is because it is 1) intuitively not as appealing, 2) more difficult to use statistically, and 3) does not explain observed results as well as the CR model.

In any event, suppose again that devices, parts, or "systems" of a given type have exhibited K modes of failure but that a particular device of this type is only subject to one (unknown before test) mode of failure. That is, there is something inherent to the particular device (not its type necessarily) which, a priori, says it can only fail by one mode. For example, under the MP model, the failure time X, of a randomly selected device for which the mode of failure is unknown, a priori, has cumulative distribution function

$$F_X(x) = P(X < x)\, p_1 + \ldots + P(X < x)\, p_K = \sum_{i=1}^{K} F_{X_i}(x)\, p_i \quad (6.3.1)$$

where p_i = probability a randomly selected device will fail by mode i.

$F_{X_i}(x)$ = the failure distribution of times to failure for mode i.

$$\sum_{i=1}^{K} p_i = 1$$

The MP model is particularly unsatisfactory because of its statistical intractability. That is, unless all $F_i(x)$ are identical both with respect to family and parameters $F_X(x)$ is not available in closed form. Thus it fails to be consistent with observed good fits to all distributions.

However, it may well be an explanation of data which is not so well behaved. Reference 185 gives examples of estimating the p_i and $F_{X_i}(x)$.

A further example is a part or device for which the production process is unstable. In relays, for example, for a time the failure problem may be one thing say spring breakage and then when this is solved it may be soft contacts, etc. The particular problem then becomes the dominant one to the (virtual) exclusion of other modes.

The MP model provides one of the few examples in accelerated life testing where statistical theory is lagging behind practice. In particular, very little has been done in estimating the $[p_i,\ F_{X_i}(x)]$ in the general case where the

$F_{X_i}(x)$ differ not only with respect to parameters but even with respect to family (normal, Weibull, etc.) of distribution and the mode of failure is not observable. If the mode of failure is observable, the p_i are generally estimable by

$$\hat{p}_i = \frac{\text{number of failures due to mode } i}{\text{total failed parts on test}} \qquad (6.3.2)$$

and then the $F_{X_i}(x)$ can be fitted easily by considering only the failure times due to mode i. The key here is whether or not the mode of failure is observable. If not, trouble begins. In either case (mode of failure observable, not observable), the expression for $F_X(x)$ is unlikely to be obtained in closed form.

6.4 Comparison of the Competing Risks (CR) and Mixed Population (MP) Models

6.4.1 Intuitive appeal, explanation of observed results.

The CR model must be rated above the MP model in this area. It is very plausible that if a given part type is subject to K modes of failure that any particular part can fail by any one (but only one) of the K modes (CR model). However, if one of the K is a dominant mode the MP and CR models will often be identical.

Regarding explanation of observed (in the literature) data results the CR model again rates the edge for it is not only consistent with data that fits say the exponential (in this case each mode is exponential with possibly differing failure rates) and the Weibull it can explain data that is not well behaved, i.e., data that does not fit any known distribution. The MP model, while explaining data which is not well behaved, is unable to explain good exponential, Weibull (or any other fit) unless all the $F_{X_i}(x)$ are identical with respect to family and parameters.

6.4.2 Tractability

There is little to choose here because unless the $F_{X_i}(x)$ are of the same family and are available in closed form, neither model is particularly tractable.

6.4.3 Ease of estimation -- Mode of failure identifiable (observable).

If the mode of failure is identifiable, the problem of estimating the $F_{X_i}(x)$ and p_i for the MP model is trivially easy since segregation of failure by mode provides estimates of $F_{X_i}(x)$ and p_i directly. For the CR model, as previously mentioned, the $F_{X_i}(x)$ are not directly observable and unless the $F_{X_i}(x)$ are all of the same family, the estimation of the $F_{X_i}(x)$ and the parameters is, so far, an unsolved statistical problem.

6.4.4 Ease of estimation -- Mode of failure not identifiable.

Excepting, the case where $K = 2$ and both $F_{X_i}(x)$ identical it is essentially an unsolved statistical problem.

7.0 BIBLIOGRAPHY

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As stated above, the bibliography includes all the literature reviewed during the study. There was logic which suggested that the bibliography should only contain articles found to relate specifically to ALT and ones that represented the foremost contributions to the state of the art. The decision was made, however, to include everything that was reviewed and allow the user of the bibliography the privilege of judging the usefulness of each article for his own particular accelerated life testing application.

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UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Hughes Aircraft Co. Ground Systems Group Fullerton CA	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP N/A

3. REPORT TITLE

ACCELERATED TESTING TECHNOLOGY

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Final Report, Feb 1966 to Jul 1967

5. AUTHOR(S) (First name, middle initial, last name)

Yurkowsky, W. Finkelstein, J. M.
Schafer, R. E.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
November 1967	420	524

8a. CONTRACT OR GRANT NO. AF30(602)-4046 b. PROJECT NO. 5519 c. Task 551902 d.	9a. ORIGINATOR'S REPORT NUMBER(S) FR 67-16-157 FR 67-16-185 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) RADC-TR-67-420, Vols. I & II

10. DISTRIBUTION STATEMENT

This document is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with the prior approval of RADC (EMERR) GAFB NY 13440.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
Donald W. Fulton, Project Engineer (EMERR)	Rome Air Development Center (EMERR) Griffiss AFB NY 13440

13. ABSTRACT

~~This~~ report, in two volumes, is devoted to an investigation into the state-of-the-art of methods for reducing the time and expense associated with life testing, particularly as it relates to parts. The areas investigated are divided into accelerated testing and more powerful statistics. The accelerated tests treated in the report are tests at stresses higher than nominal design levels applied either singly or in combinations at constant, progressivly increasing, or increasing by steps, stress levels. The more powerful statistical approaches found to be pertinent are those using prior information distribution-dependent methods and distribution-free methods.

Volume one ~~of this report~~, prepared in handbook format, gives an assessment of advantages and limitations of the present available methods for reducing test time and expenses and presents in compact, but complete form, the instructions for using the methods which represent the state-of-the-art of accelerated life testing.

Volume two presents the methodology used in performing the study, explains the evaluation systems used on the methods reported in the literature, establishes the criteria for inclusion of a method in Volume one, and highlights to useful methods developed as well as those which show promise for future development.

DD FORM 1 NOV 65 1473

UNCLASSIFIED

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Accelerated Testing						
Reliability Demonstration						
Statistics						